Education In Human Values Through Mathematics

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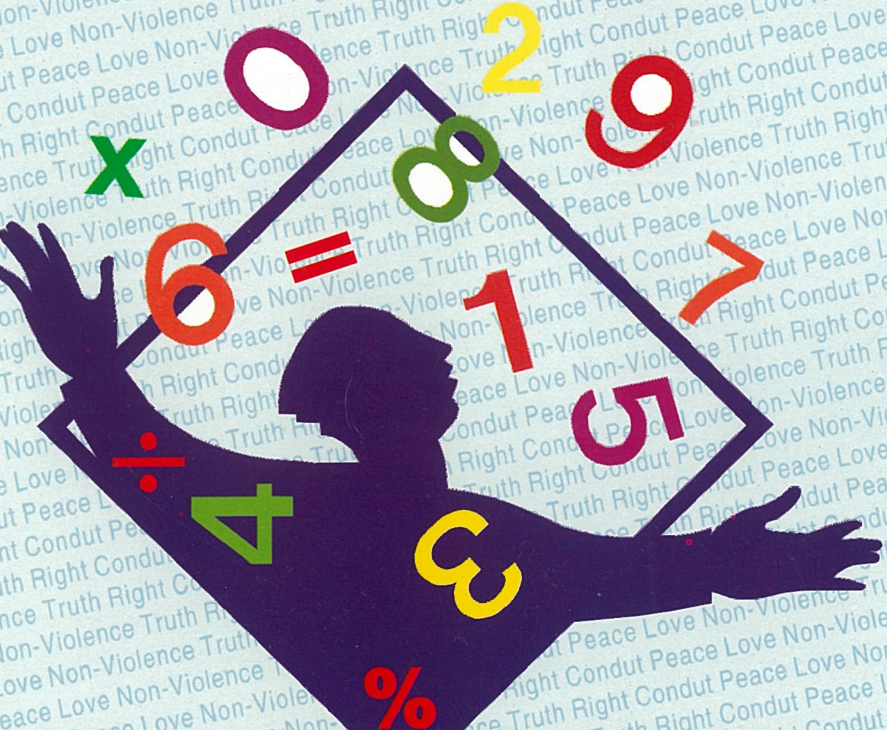

Margaret Taplin

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By

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Foreword

In teaching children at school, the Sathya Sai Education in Human Values specifies that human values are integrated into all subjects as well as in all extra-curricular activites. In case of morality or language classes, the Direct Method is recommended where all the five techniques are used. These five techniques are: silent sitting or dynamic thoughts, story telling, group singing and group activities. However, when other subjects are taught, then values are integrated.

Many teachers embarking on the Sathya Sai Education in Human Values for the first time may find some difficulty to integrate values into all subjects especially in science and mathematics. There is lack of books for teachers, which will explain how to integrate values. Margaret Taplin has written one of the first books, which deals with human values in mathematics. It is a valuable book for teachers to develop their own lesson plans that will inculcate values in Mathematics. This book also contains human values that can be learnt from examples of great mathematicians. There is also a wealth of references for further reading and research.

This is only the beginning. It is hoped that this book will stimulate teachers who are using Sathya Sai Education in Human Values to write more books from their own experiences that will help other teachers to integrate human values into various subjects - science, geography, history, arts, music, sports etc....

Dr. Art-Ong Jumsai
Director
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Introduction

Knowledge without character is not merely useless, but positively dangerous. Academic knowledge alone is of no great value. It may help one to earn a livelihood, but education should go beyond the preparation of earning a living. It should prepare one for the challenges of life, morally and spiritually. Education has not affected habits and conduct, daily behaviours and inter-personal contacts. Humility, detachment, discrimination, eagerness to serve others, reverence, renunciation - such virtues are absent amongst the educated. Raising the standard of living must also mean raising ethical, moral and spiritual standards. Then only can education lead to progress in human values and harmony in social life. The promotion of human values must become an integral part of the education process. It is because students today have not acquired human values that they are often behaving like demons. Human values are in everyone. What we need are persons who will provide the stimulus and the encouragement to bring them out. Modern education develops the intellect and imparts skills, but does not promote good qualities in any way. Of what value is the acquisition of all the knowledge in the world, if there is no character? Education in Human Values is based on the five universal values of truth, right conduct, peace, love and non-violence, corresponding to the intellectual, physical, emotional, psychic and spiritual aspects of the child. Science and technology have made astonishing progress but humanity is going on the downward path. The end of education is character. We must learn to see good in everything, everywhere, at all times. Our life should be like a light, shining with the rays of peace, love, right-conduct, truth and non-violence. All the time, we must be alert, keeping the conscious mind disciplined.... What the head thinks should be examined critically by the heart and the right decisions should be carried out by the hands. This should be the primary product of education. Education should go beyond the preparation for earning a living. It should prepare one for the challenges of life morally and spiritually. It is because human values are absent in "educated" persons that we find them steeped in anxiety and worry.

Man's achievements in the fields of science and technology have helped to improve the material conditions of living. What we need today, however, is a transformation of the spirit. Education should serve not only to develop one's intelligence and skills, but also help to broaden one's outlook and make one useful to society and the world at large.

True education should make a person compassionate and humane.

The way to win over the emotions within us is to search for peace. Peace can only be obtained when we have learned self-control, when we have stilled our mind.

In the 1920s, H.G. Wells made a very prophetic statement. He said: "History is increasingly becoming a race between education and catastrophe!" ....What needs to be examined are the priorities and goals of education.

This is the apathy of education today. It teaches us languages and concepts but it does not teach us how to live in a balanced way with outselves and others.

Education is for life not merely for making a living.

The main emphasis in education today is on intellectual achievement, in other words, passing exams and getting good marks. There is little or no attention put on the moral and spiritual development of the child. The result of this is that bright children may make the grade academically, but they are not well equipped to put the knowledge that they have learned to good use for their own evolvement and for the benefit of society.

The basic aim of the Sathya Sai Education in Human Values (SSEHV) programme is the development of character, raising the consciousness by developing all the five layers of the human personality: intellectual, physical, emotional, psyche and spiritual. The programme began in India , and is now active in 42 countries around the world, including Thailand, Malaysia, the United Kingdom, Australia, Hong Kong, and many countries in the continents of South America and Africa.

Where time and curriculum constraints can often make it difficult to include Education in Human Values as an additional topic in the curriculum, there are many opportunities to teach its principles through existing subjects and topics. Rather than treating values education only as a subject in its own right, this kind of integration has the advantage that schools do not have to abdicate in any way their responsibility to teach the academic skills but simply that they will be rethinking the ways in which they do this. Teachers often mention that they find it relatively easy to draw out values in humanities subjects but that they find it more difficult to do so in mathematics. The purpose of this book, therefore, is to suggest some ways in which values can be drawn out of existing mathematics topics and teaching approaches.

In generating the activities described in this book, attempts have been made to integrate the five universal human values as shown in the table below:

Five universal values and examples of sub-values

Truth:	accuracy, curiosity, discrimination, honesty, human understanding, integrity, self-reflection, sincerity
Right Action:	courage, dependability, determination, efficiency, endurance, healthy living, independence, initiative, perseverance
Peace:	calmness, concentration, contentment, equanimity, optimism, self-acceptance, self-discipline, self-esteem
Love:	compassion, consideration, forgiveness, humaneness, interdependence, selflessness, tolerance
Non-violence:	benevolence, co-operation, concern for ecological balance, respect for diversity, respect for life, respect for property

Attention has also been paid to the questions of whether the activities described contribute to:

- √ bringing out human excellence at all levels: character, academic, and "being";
- √ the all-round development of the child (the heart as well as the head and the hands);
- √ helping children to know who they are;
- √ helping children to realise their full potential;
- √ developing self-reliance, self-confidence, and attitudes of selfless service

The book has been compiled to address the needs of different people in different ways. First it is intended as a resource for teachers who are already teaching education in human values. For these people, it is designed to show some strategies for integrating some of the ideas of the SSEHV programme into the regular mathematics programme. Second, the book is intended to make other teachers, who may not be familiar with the SSEHV programme, aware that it is possible to teach pupils about human values without making any particular additions to their regular teaching programme. Third, the book seeks to cater for workshop and discussion group leaders, by contributing ideas for discussion and action research in the classroom. The fourth aim is to cater for teachers and student teachers who wish to read in greater depth about current research and teaching theories which can reflect SSEHV issues. It is not simply a "recipe book" of teaching ideas, but will help teachers to help themselves in their programme development, by providing ideas, starting points, and useful references.

The book has been divided into four sections:

Educating for Human Values Through Approaches to Teaching Mathematics,

Using Mathematics as a Tool to Practise Human Values,

Teaching Human Values Through Examples of Great Mathematicians,

Mathematics and "Silent Sitting".

For further teaching ideas, articles and teachers' reflections about the integration of education in human values into mainstream schools, please visit www. ssehv.org.

Educating For Human Values Through Approaches To Teaching Mathematics

The purpose of this section is to illustrate some ways in which current theories about approaches to teaching mathematics can expose pupils to the ideas of the EHV programme. Each chapter begins with a collection of quotations directly from Sathya Sai Baba or indirectly from other EHV materials. The sources of these quotations have not specifically been acknowledged because they appear in similar form in many different places, but the quotations have been printed in *italics* at the beginning of each section.

This section will explore issues including:

the development of *general knowledge and common sense* through different types of problem-solving activities and the fostering of mathematical thinking skills;

perseverance - what affects it, how it can be enhanced through mathematics activities, and strategies for helping pupils to increase their perseverance;

creating unity - how co-operative grouping and discussion in the mathematics classroom can teach pupils to be more tolerant and caring, and to work together to achieve common goals;

understanding and accepting *differences between individuals*, and helping minority groups to achieve their potential.

The first part of each chapter gives a brief description of the teaching approaches, and the way they can reflect human values. It is designed for busy teachers who do not have time for more detailed reading. The next part suggests some discussion or classroom based action research questions. Teachers are invited to explore answers to some of these questions and discuss the outcomes with their colleagues.

The third part of each chapter is theoretical and research-based, designed to give more detailed information for student teachers and teacher-researchers. It describes recent research which supports the particular teaching approach under consideration.

Using Mathematics as a Tool to Practise Human Values

There are many ways in which the use of mathematics can help to reinforce and apply important human values. This section describes some sample classroom activities that enable pupils to realise what a powerful tool mathematics can be. The particular emphasis is on the way in which teachers can use topics in the current mathematics syllabus to facilitate the practise of human values, without having to introduce extra topics into the curriculum. Topic areas include:

> *learning to conserve and protect the environment* through, for example, monitoring the use of paper, or investigating the potential to recycle some materials (statistics, graphing);
>
> creating awareness of social issues such as money management (problem solving, simple and compound interest, basic operations), problems such as gambling (probability), and sharing food (measurement, estimation, calculations with whole numbers and fractions);
>
> understanding their heritage and culture through learning more about the history of how mathematics developed, learning about different ways of thinking about mathematics in different cultures, and appreciating the balance and beauty of mathematics (patterns and sequences, spatial activities).

This section will be supplemented by a volume of lesson plans which will be collected from EHV teachers on an ongoing basis and updated frequently.

Teaching Human Values Through Examples of Great Mathematicians

This section offers brief biographies of famous mathematicians, past and present, who have *proclaimed the supremacy of morality and character...refraining from harming others, showing compassion, courage, sacrifices*. It is intended that teachers will share these biographical excerpts with their pupils in conjunction with the regular study of the topics with which these mathematicians are associated. A reference list gives further information about biographies which teachers can use for their own reference, and those which are suitable references for children's project work.

Mathematics and "Silent Sitting"

This section introduces a simple technique fundamental to Sathya Sai Education in Human Values that enables children to make optimum use of all parts of their brains for doing mathematics. Some research evidence is presented that indicates the positive effects of this technique on studens' concentration, behaviour and achievement, and some examples are given to stimulate teachers to develop their own.

CONTENTS

SECTION 1

Educating For Human Values Through Approaches To Teaching Mathematics

CHAPTER 1

Teaching Values Through A Problem-Solving Approach To Mathematics

It may help one to earn a livelihood, but education should go beyond the preparation of earning a living. It should prepare one for the challenges of life.

Bookish knowledge alone is not enough. It is superficial and not practical. Students need also general knowledge and common sense.

Science alone is not enough. There must be discrimination for utilising the discoveries of science for right purposes.

Education should serve to develop powers of discrimination and foster the sense of patriotism so that the educated may engage themselves in service to society.

Education today is a process of filling the mind with the contents of books, emptying the contents in the examination hall and returning empty-headed....What you learn should become a part of your whole being. Only then will you have a sense of fulfilment, and establish complete harmony in thought, word and deed. Today, the country needs persons who lead such integral lives.

When one's interest is rooted in some field of knowledge, attention on it becomes firm and memory will enshrine it in the mind....Steady interest is essential in order to master worthy knowledge.

Do not turn them into experts in mathematics, unable to add up a simple domestic bill; scholars in the geography of America, but unable to direct a pilgrim who desires to know in which direction Westminster Cathedral or the London Mosque lies; prodigies in Algebra who are helpless when asked to define the area of their own rooms...

Education should serve not only to develop one's intelligence and skills, but also help to broaden one's outlook and make him useful to society and the world at large.

The quotations above are concerned with the following values:

- equipping students to meet the challenges of life,
- developing general knowledge and common sense,
- learning how to be discriminating in use of knowledge, that is to know what knowledge is appropriate to use for what purposes,
- integrating what is learned with the whole being,
- arousing attention and interest in the field of knowledge so it will be mastered in a worthy way.

Traditionally, mathematics teaching has been very much concerned with filling pupils' heads with rules and knowledge, to be remembered until the examination, and then forgotten by all but those few who need to use the knowledge in their work. But, more recently, educators have come to realise that mathematics teaching does not have to be like this. By re-thinking the way we teach mathematical topics, we can help students to develop the values of common sense and discriminatory use of knowledge, arouse their interest in the subject to a level where it *can* become integrated with the whole being, and help them to be able to use their mathematics knowledge as a tool for meeting the challenges of life.

This chapter will talk about developing the above values by teaching mathematics via a problem-solving approach. The first section will explain, in more detail, how and why the use of problem solving can enhance these values. The second section will give some practical suggestions for using a problem-solving approach to teach mathematics.

How Can Values be Enhanced by Teaching Mathematics via Problem Solving?

Increasing numbers of individuals need to be able to think for themselves in a constantly changing environment, particularly as technology is making larger quantities of information easier to access and to manipulate. They also need to be able to adapt to unfamiliar or unpredictable situations more easily than people needed to in the past. Teaching mathematics encompasses skills and functions which are a part of everyday life.

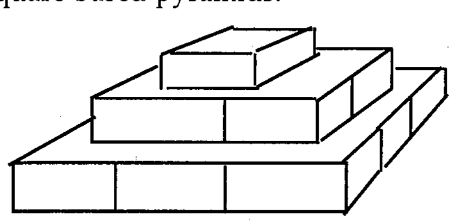
Examples

reading a map to find directions
understanding weather reports
understanding economic indicators
understanding loan repayments
calculating whether the cheapest item is the best buy

Presenting a problem and developing the skills needed to solve that problem is more motivational than teaching the skills without a context. It allows the students to see a reason for learning the mathematics, and hence to become more deeply involved in learning it.

Example of a Concept Taught Through a Problem-Solving Context

Ms. Chan wanted to teach her class the concept of square numbers. She began by telling them a story about the Egyptian pyramids. She told them that the Great Pyramid of Cheops is 150 metres high, and is built of over 2 000 000 stone blocks each weighing about 2.5 tonnes, and took 100 000 men 20 years to complete.
Next, Ms. Chan gave each group of students 276 small cubes and asked them to use all of the cubes to build their own square-based pyramids.



When they had done this, she asked them whether they noticed any pattern in the numbers of cubes which made up each layer. She asked them to predict how many cubes would be used for the tenth layer, the 100th layer, etc. and to test their predictions.

Students' Comments About Learning Mathematics Through Problem Solving

One particular high school teacher had a problem solving approach in his classroom and I will always remember the excitement and the "Aha" experiences once a problem was solved. It was probably this teacher who made me achieve the highest mathematical level possible during College.

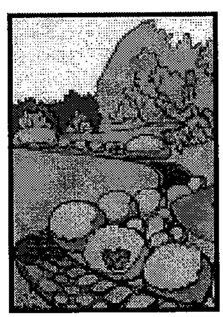
Student Teacher

I personally enjoyed the problem solving sessions we had during our maths course this year. Discovering things for yourself is far more rewarding and enjoyable than being lectured at. In the latter case, I feel that a lot of information goes in one ear and out the other. The problem solving approach allows me to remember things a lot better, which is most probably due to the "Aha" experience when you see a solution, or something clicks into place.

High School Student

Teaching through problem solving can enhance logical reasoning, helping people to be able to decide what rule, if any, a situation requires, or if necessary to develop their own rules in a situation where an existing rule cannot be directly applied.

A group of twelve-year-olds was given the following diagram, and asked to calculate how many one-metre square paving stones would be needed to make a two-metre wide path around the garden, which is 9 metres wide and 13 metres long.



The pupils did not immediately know a formula to solve this problem. They needed to decide which of the formulae they *did* know, such as perimeter and area, could be used, and they also needed to decide *how* to use this information.

Problem solving can allow the whole person to develop by experiencing the full range of emotions associated with various stages of the solution process.

Examples

The problem that we worked on today had us make a hypothesis. Through testing, our hypothesis was proven incorrect. The problem solving approach allowed our group to find this out for ourselves, which made the "bitter pill" of our mistake easier to follow.

I found this activity to be quite a challenge. I felt intimidated because I could not see an immediate solution,and wanted to give up. I was gripped by a feeling of panic. I had to read the question many times before I understood what I had to find. I really had to "dig down" into the depths of my memory to recall the knowledge I needed to solve the problem.

> Seeing patterns develop before my own eyes was a powerful experience: it had a stimulating effect. I felt that I had to explore further in a quest for an answer., and for more knowledge.
>
> *Extracts from a student teacher's journal after three separate problem solving sessions*

The student who wrote the extracts above, has illustrated how *interest rooted in the problem* encouraged *steady interest needed to master worthy knowledge.* Experience with problem solving can develop curiosity, confidence and open-mindedness.

How To Teach Human Values By Incorporating Problem Solving Into The Mathematics Programme

The first section of this chapter explained why problem solving is an important vehicle for educating students for life by promoting interest, developing common sense and the power to discriminate. In particular, it is an approach which encourages flexibility, the ability to respond to unexpected situations or situations that do not have an immediate solution, and helps to develop perseverance in the face of failure (which will be discussed in more detail in the next chapter). This section will describe the types of problem solving which can be used to enhance these characteristics, and will give some suggestions of how it can be used in the mathematics programme.

It is important to remember that what is a problem for one student is not necessarily so for another. A situation becomes a problem when the individual cannot immediately resolve it, but needs to think of the information available and the way it can be used to arrive at a solution.

Problem solving is an **approach to teaching** rather than a topic, but students do need to be equipped with certain strategies to be able to respond to this type of approach. Problem investigations can be long-term projects or short-term, and can be aimed at individuals, small groups, or the whole class.

There are three types of problems to which students should be exposed:

(i) word problems, where the concept is embedded in a real-world situation and the student is required to recognise and apply the appropriate algorithm/rule *preparing pupils for the challenges of life);*

(ii) non-routine problems which require a higher degree of interpretation and organisation of the information in the problem, rather than just the recognition and application of an algorithm *(encouraging the development of general knowledge and common sense),*

(iii) "real" problems, concerned with investigating a problem which is real to the students, does not necessarily have a fixed solution, and uses mathematics as a tool to find a solution *(engaging pupils in service to society).*

Each of these problem types will be described in more detail below.

Problems which require the direct use of a mathematics rule or concept

By solving these types of problems, students are learning to *discriminate what knowledge is* *required for certain situations, and developing their common sense.* The following examples have been adapted from the HBJ Mathematics Series, to show how values such as sharing, helping and conserving energy can be included in the wording of the problems. They increase in difficulty as they require more steps:

Examples

7 children went mushrooming and agreed to share. They picked 245 mushrooms. How will they find out how many they will get each?

Nick helps his elderly neighbour for 1/4 of an hour every week night and for 1/2 an hour at the weekend. How much time does he spend helping her in 1 week?

Recently it was discovered that a clean engine uses less fuel. An aeroplane used 4700 litres of fuel. After it was cleaned it was found to use 4630 litres for the same trip. If fuel cost 59 cents a litre, how much more economical is the clean plane?

Sometimes it is important to give problems which contain too much information, so the pupils need to select what is appropriate and relevant:

Examples

Last week I travelled on a train for a distance of 1093 kilometres. I left at 8 a.m. and averaged 86 km/hour for the first four hours of the journey. The train stopped at a station for 1 1/2 hours and then travelled for another three hours at an average speed of 78 km/hour before stopping at another station. How far had I travelled?

To be able to solve these problems, the pupils cannot just use the *bookish knowledge* which they have been taught. They also need to apply *general knowledge and common sense.*

Another type of problem, which will encourage pupils to be *resourceful*, is that which does not give enough information. These problems are often called Fermi problems, named after the mathematician who made them popular. When people first see a Fermi problem they immediately think they need more information to solve it. Basically though common sense and experience can allow for reasonable solutions. The solution of these problems relies totally on knowledge and experience which the students already have. They are problems which are non-threatening, and can be solved in a co-operative environment. These problems can be related to social issues, for example:

Examples

How many litres of petrol are consumed in your town in a day?

How much money would the average person in your town save in a year by walking instead of driving or taking public transport?

How much food is wasted by an average family in a week?

Using a Fermi Problem to Promote Human Values

Ms. Lam wanted to teach her class of ten-year-olds about the value of money, and to appreciate what their parents were doing for them:

"I believe that students should be aware of this important issue and thus can be more considerate when a money issue raised in their own family, such as failure to persuade their parents to by an expensive present. In solving the problems, I think that students can have a better understanding of the concept of money, not simply as a tool of buying and selling things.

"First I told the class a story about Peter's argument with his family. Peter failed to persuade his parents to buy expensive sportshoes as his birthday present and thought that his parents did not treat him well. The parents also felt upset as they regarded this son as an inconsiderate child. They thought that he should understand that the economy is not so good. They asked Peter if he knew about how much money was being spent on him throughout the whole year. Unfortunately, Peter could not produce the answer immediately. So I asked the class if they could help Peter. I asked them to find answers to the following problems:

How much money do your parents spend on you in a year?
How much money have your parents spent on you up till now?
How much money will your parents have spent on you by the time you finish secondary school?

How much money will be spent on raising children in the whole country this year?

"The students were formed into groups of 4 to find out the possible data that they need to know. Later, the groups were asked to present their data and the way of finding out the answer. Finally, I concluded that this is an open question as each person may have different expenditure along with some common human basic needs such as food, clothes and travelling fares. Anyway, the answer should be regarded as a large sum of money and thus give them a better understanding of their parents' burden."

Sometimes pupils can be asked to make up their own problems, which can help to enhance their understanding. This can encourage them to be *flexible,* and to realise that there can be *more than one way of looking at a problem.* To bring out the focus on education in human values, they can be asked to relate their problems to a particular value, such as honesty, sharing, persistence or caring for the environment.

Example
32 divided by 5

Make up a problem for which the answer is 6.4: I made 32 biscuits to give to some hungry children. 5 children came to eat them. How many should I give to each child?

Make up a problem for which the answer is 7: 32 children are going on a trip. If 5 children can travel in each car, how many cars will be needed?

Make up a problem for which the answer is 6: If I have 32 metres of wood to make shelves each 5 metres long, how many shelves can I make?

Non-Routine Problems

Non-routine problems can be used to encourage logical thinking, reinforce or extend pupils' understanding of concepts, and to develop problem-solving strategies which can be applied to other situations. The following are examples of non-routine problems:

Examples

By changing six figures into zeros you can make this sum equal 1111.

111
333
555
777
<u>999</u>
<u>2775</u>

What is my mystery number?
If I divide it by 3 the remainder is 1.
If I divide it by 4 the remainder is 2.
If I divide it by 5 the remainder is 3.
If I divide it by 6 the remainder is 4.

Real Problem Solving

Bohan, Irby and Vogel (1995) suggest a seven-step model for doing research in the classroom, to enable students to become "producers of knowledge rather than merely consumers" (p.256).

Step 1: What are some questions you would like answered?
The students brainstorm to think of things they would like to know, questions they would like to answer, or problems that they have observed in the school or community. Establish a rule that no one is to judge the thoughts of another. If someone repeats an idea already on the chalkboard, write it up again. Never say, "We already said that," as this type of response stifles creative thinking.

Step 2: Choose a problem or a research question.
The students were concerned with the amount of garbage produced in the school cafeteria and its impact on the environment. The research question was, "What part of the garbage in our school cafeteria is recyclable?"

Step 3: Predict what the outcome will be.

Step 4: Develop a plan to test your hypothesis.
The following need to be considered: Who will need to give permission to collect the data? Courtesy - when can we conveniently discuss this project with the cafeteria manager? Time - how long will it take to collect the data? Cost - will it cost anything? Safety - what measures must we take to ensure safety?

Step 5: Carry out the plan:
Collect the data and discuss ways in which the students might report the findings (e.g. graphs)

Step 6: Analyse the data: did the test support our hypothesis?
What mathematical tools will be needed to analyse the data: recognising the most suitable type of graph; mean; mode; median?

Step 7: Reflection.
What did we learn? Will our findings contribute to our school, our community, or our world? How can we share our findings with others? If we repeated this experiment at another time, or in another school, could we expect the same results? Why or why not? Who might be interested in our results? "The final thought to leave with students is that they can be researchers and producers of new information and that new knowledge can be produced and communicated through mathematics. Their findings may contribute to the knowledge base of the class, the school, the community, or society as a whole. *Their findings may affect their school or their world in a very positive way*" (Bohan et al., 1995, p.260).

Some examples of real problems which can allow students to become more aware of human values will be given in Part II of this book.

Mathematical Investigations

Mathematical investigations can fit into any of the above three categories. These are problems, or questions, which often start in response to the pupils' questions, or questions posed by the teacher such as, "Could we have done the same thing with 3 other numbers?", or, "What would happen if...." (Bird, 1983). At the beginning of an investigation, the pupils do not know if there will be a suitable answer, or more than one answer. Furthermore, the teacher either does not know the outcome, or pretends not to know. Bird suggests that an investigation approach is suitable for many topics in the curriculum and encourages communication, confidence, motivation and understanding as well as mathematical thinking. The use of this approach makes it difficult for pupils to just carry out routine tasks without thinking about what they are doing.

An example is an investigation designed to help students to discover the area = length x width rule:

Example

Draw several rectangles. For each one, count the number of squares long and the number of squares wide. Count the number of squares in the area.

length	width	area

Can you find a connection between the length, width and area?
Will this always happen?
Can you explain why?

Bird (1983) believes that investigational problem solving can be enhanced if students are encouraged to ask their own questions. She suggested that the teacher can introduce a "starter" to the whole class, ask the students to work at it for a short time, ask them to jot down any questions which occurred to them while doing it, and pool ideas. Initially it will be necessary for the teacher to provide some examples of "pooled" questions, for example:

Does it always work?
Is there a reason for this happening?

How many are there?
Is there any connection between this and.....?

The pupils can be invited to look at each other's work and, especially if they have different answers, to discuss "who is right".

The following example of pupils generating their own theories is from Bird (1983, p.36).

Quick tests of Divisibility

Some of the pupils tried to find a quick test of divisibility by 6. Here are some snippets of their attempts.

To see if the number will divide by 6. The number would have to be an even and then when you add the digits up then they must be able to divide by 3, e.g.
7 5 3 4 2
7+5+3+4+2 = 21
2+1 = 3
or
7x3 = 21
and just to make sure it works, 6 x 12 557 = 75 342

add last two numbers together and if there is a 0 go on to the next number
(Jonathon)

Strategies for Problem Solving

As mentioned earlier, one of the main reasons for teaching mathematics through a problem-solving approach is to equip pupils with a wide range of strategies which they can use whenever they encounter unfamiliar problem situations. McDonough (1984) lists a number of alternatives which pupils can try when they become "stuck":

STUCK?
look for a pattern ⇨ use or make a picture, objects or graph
make an organised list, table or chart ⇨ guess and check
restate the problem ⇨ work backwards
solve a similar problem first ⇨ write down important information
experiment or act out the problem ⇨ look for key words and phrases
write out an equation ⇨ change your point of view
check for hidden assumptions ⇨ onsider all possibilities

McDonough encouraged children to write their strategies on the backs of cards on which the problems were written, so they could be shared and discussed with their classmates. The following is an example of one such problem (p.62).

Joe is a fast cook. It takes him three minutes to boil an egg in a pot designed for twelve eggs. How long does it take him to boil six eggs?	Strategies Written on Card by Pupils. ♦ Restate the problem ♦ List the information which has been given ♦ Refer to a similar problem: ♦ If it takes 20 minutes to dry one teatowel how long does it take to dry five teatowels? ♦ Draw an analogy e.g. with roast potatoes

For students who are still having difficulties, McDonough advised that the teacher:

- ask the pupils to restate the problem in their own words and if this indicates that they have mis-read or mis-interpreted the card, ask them to read the instructions again,
- to help with the understanding of the written instructions question the pupils carefully to find out if they know the meanings of particular words and phrases (i.e. mathematical terminology),
- have the pupils show the teacher what they have done, compare this to what is asked in the instructions, and question the pupils to see if they could think of another method, for example, "Could you have done this another way?" or, "Have you ever done a task like this before?"
- if necessary, give the children a small hint but only after questioning them carefully to find out what stage they have reached.

If the teacher follows procedures such as those described above, the pupils will be encouraged to be more thoughtful and self-reliant.

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Some Questions For Discussion With Colleagues, Or Action Research In Your Classroom

In this chapter, it has been shown how mathematical problem solving can contribute to the following values:
perseverance
thoughtfulness
self-reliance
resourcefulness
common sense
appreciation of social issues.

Use a problem-solving approach to teach a standard mathematics topic, and observe the extent to which the students show signs of these values. Discuss your findings with your students.

Monitor the range of emotions which students experience during a problem-solving experience. How can these contribute to the development of the student's character? Discuss this with your students.

Present a standard topic in a "real-life" context, as in the example above. Compare the level of the students' interest and involvement, compared to when the same topic is presented in a more traditional way.

Over time (e.g. a whole school year), observe changes in the pupils' ability to be able to transfer knowledge to unfamiliar situations.

"Real Problem Solving" enables pupils to practise their mathematics skills or learn new ones, by using mathematics as a tool to address a problem. Have your pupils identify and explore a solution to a problem. Identify the mathematical skills which are used, and the values which are encouraged. Discuss with your pupils how their project has been useful in helping others.

Further Reading For The Student Or Researcher

In the past, while society appeared to need a large number of people who could correctly use mathematical procedures and only a few who understood the processes involved, there was little need to question the basic traditional programme or the rote procedures used in schools to teach mathematics. It was sufficient that society could depend upon the few people who were able to develop an understanding for themselves.

Over recent years, however, the traditional model of mathematics, with emphasis on the direct application of a rote-learned rule to get a correct answer, has become less acceptable to society. This has been due to changes in many factors, some technological, some social. For example, the technology of the calculator and the computer have made redundant the need for learning algorithms to obtain answers to certain standard questions. This applies even at the secondary school level, with calculators able to produce such algorithms as those for solving simultaneous equations or graphing relationships. Also, the changing social values of education, becoming more centred on the child and emphasising individual development, and the development of thinking skills, have shifted the focus criterion in all disciplines from 'the answer' to the way in which the process of obtaining the answer contributes to the individual's total development. This involves not only specific skills for living but also, increasingly, aesthetic enjoyment and an understanding of how things work. The importance of being able to think for oneself in a constantly changing environment, for example, emphasises the need to understand the development of the processes required in reaching a solution (Clarke and McDonough, 1989). Becoming a mathematical problem solver is an essential pre-requisite for becoming a productive citizen (Romberg, 1994).

WHAT IS A 'PROBLEM-SOLVING APPROACH'?

As the emphasis has shifted from teaching problem solving to teaching *via* problem solving (Lester, Masingila, Mau, Lambdin, dos Santon and Raymond, 1994), many writers have attempted to clarify what is meant by a problem-solving approach to teaching mathematics. The focus is on teaching mathematical topics through problem-solving contexts and enquiry-oriented environments which are characterised by the teacher 'helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying' (Lester et al., 1994, p.154). Specific characteristics of a problem-solving approach include:

interactions between students/ students and teacher/students (Van Zoest et al., 1994),

mathematical dialogue and consensus between students (Van Zoest et al., 1994),

teachers providing just enough information to establish background/intent of the problem, and students clarifing, interpreting, and attempting to construct one or more solution processes (Cobb et al., 1991),

teachers guiding, coaching, asking insightful questions and sharing in the process of solving problems (Lester et al., 1994),

teachers knowing when it is appropriate to intervene, and when to step back and let the pupils make their own way (Lester et al., 1994).

A further characteristic is that a problem-solving approach can be used to encourage students to make generalisations about rules and concepts, a process which is central to mathematics (Evan and Lappin, 1994).

Schoenfeld (in Olkin and Schoenfeld, 1994, p.43) described the way in which the use of problem solving in his teaching has changed since the 1970s:

> My early problem-solving courses focused on problems amenable to solutions by Polya-type heuristics: draw a diagram, examine special cases or analogies, specialize, generalize, and so on. Over the years the courses evolved to the point where they focused less on heuristics per se and more on introducing students to fundamental ideas: the importance of mathematical reasoning and proof..., for example, and of sustained mathematical investigations (where my problems served as starting points for serious explorations, rather than tasks to be completed).

Schoenfeld also suggested that a good problem should be one which can be extended to lead to mathematical explorations and generalisations. He described three characteristics of mathematical thinking:

valuing the processes of mathematization and abstraction and having the predilection to apply them,

developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure - mathematical sense-making (Schoenfeld, 1994, p.60).

As Cobb et al. (1991) suggested, the purpose for engaging in problem solving is not just to solve specific problems, but to 'encourage the interiorization and reorganization of the involved schemes as a result of the activity' (p.187). Not only does this approach develop students' confidence in their own ability to think mathematically (Schifter and Fosnot, 1993), it is a vehicle for students to construct, evaluate and refine their own theories about mathematics and the theories of others (NCTM, 1989). Because it has become so predominant a requirement of teaching, it is important to consider the processes themselves in more detail.

The Role Of Problem Solving In Teaching Mathematics As A Process

Problem solving is an important component of mathematics education because it is the single vehicle which seems to be able to achieve at school level all three of the values of mathematics listed at the outset of this article: functional, logical and aesthetic. Let us consider how problem solving is a useful medium for each of these.

It has already been pointed out that mathematics is an essential discipline because of its practical role to the individual and society. Through a problem-solving approach, this aspect of mathematics can be developed. Presenting a problem and developing the skills needed to solve that problem is more motivational than teaching the skills without a context. Such motivation gives problem solving special value as a vehicle for learning new concepts and skills or the reinforcement of skills already acquired (Stanic and Kilpatrick, 1989, NCTM, 1989). Approaching mathematics through problem solving can create a context which simulates real life and therefore justifies the mathematics rather than treating it as an end in itself. The National Council of Teachers of Mathematics (NCTM, 1980) recommended that problem solving be the focus of mathematics teaching because, they say, it encompasses skills and functions which are an important part of everyday life. Furthermore it can help people to adapt to changes and unexpected problems in their careers and other aspects of their lives. More recently the Council endorsed this recommendation (NCTM, 1989) with the statement that problem solving should underly all aspects of mathematics teaching in order to give students experience of the power of mathematics in the world around them. They see problem solving as a vehicle for students to construct, evaluate and refine their own theories about mathematics and the theories of others.

According to Resnick (1987) a problem-solving approach contributes to the practical use of mathematics by helping people to develop the facility to be adaptable when, for instance, technology breaks down. It can thus also help people to transfer into new work environments at this time when most are likely to be faced with several career changes during a working lifetime (NCTM, 1989). Resnick expressed the belief that 'school should focus its efforts on preparing people to be good adaptive learners, so that they can perform effectively when situations are unpredictable and task demands change' (p.18). Cockcroft (1982) also advocated problem solving as a means of developing mathematical thinking as a tool for daily living, saying that problem-solving ability lies 'at the heart of mathematics' (p.73) because it is the means by which mathematics can be applied to a variety of unfamiliar situations.

Problem solving is, however, more than a vehicle for teaching and reinforcing mathematical knowledge and helping to meet everyday challenges. It is also a skill which can enhance logical reasoning. Individuals can no longer function optimally in society by just knowing the rules to follow to obtain a correct answer. They also need to be able to decide through a process of logical deduction what algorithm, if any, a situation requires, and sometimes need to be able to develop their own rules in a situation where an algorithm cannot be directly applied. For these reasons problem solving can be developed as a valuable skill in itself, a way of thinking (NCTM, 1989), rather than just as the means to an end of finding the correct answer.

Many writers have emphasised the importance of problem solving as a means of developing the logical thinking aspect of mathematics. 'If education fails to contribute to the development of the intelligence, it is obviously incomplete. Yet intelligence is essentially the ability to solve problems: everyday problems, personal problems ... '(Polya, 1980, p.1). Modern definitions of intelligence (Gardner, 1985) talk about practical intelligence which enables 'the individual *to resolve genuine problems or difficulties* that he or she encounters' (p.60) and also encourages the individual to find or create problems 'thereby laying the groundwork for the acquisition of new knowledge' (p.85). As was pointed out earlier, standard mathematics, with the emphasis on the acquisition of knowledge, does not necessarily cater for these needs. Resnick (1987) described the discrepancies which exist between the algorithmic approaches taught in schools and the 'invented' strategies which most people use in the workforce in order to solve practical problems which do not always fit neatly into a taught algorithm. As she says, most people have developed 'rules of thumb' for calculating, for example, quantities, discounts or the amount of change they should give, and these rarely involve standard algorithms. Training in problem-solving techniques equips people more readily with the ability to adapt to such situations.

A further reason why a problem-solving approach is valuable is as an aesthetic form. Problem solving allows the student to experience a range of emotions associated with various stages in the solution process. Mathematicians who successfully solve problems say that the experience of having done so contributes to an appreciation for the 'power and beauty of mathematics' (NCTM, 1989, p.77), the "joy of banging your head against a mathematical wall, and then discovering that there might be ways of either going around or over that wall" (Olkin and Schoenfeld, 1994, p.43). They also speak of the willingness or even desire to engage with a task for a length of time which causes the task to cease being a 'puzzle' and allows it to become a problem. However, although it is this engagement which initially motivates the solver to pursue a problem, it is still necessary for certain techniques to be available for the involvement to continue successfully. Hence more needs to be understood about what these techniques are and how they can best be made available.

In the past decade it has been suggested that problem-solving techniques can be made available most effectively through making problem solving the focus of the mathematics curriculum. Although mathematical problems have traditionally been a part of the mathematics curriculum, it has been only comparatively recently that problem solving has come to be regarded as an important medium for teaching and learning mathematics (Stanic and Kilpatrick, 1989). In the past problem solving had a place in the mathematics classroom, but it was usually used in a token way as a starting point to obtain a single correct answer, usually by following a single 'correct' procedure. More recently, however, professional organisations such as the National Council of Teachers of Mathematics (NCTM, 1980 and 1989) have recommended that the mathematics curriculum should be organized around problem solving, focusing on:

(i) developing skills and the ability to apply these skills to unfamiliar situations,

(ii) gathering, organising, interpreting and communicating information,

(iii) formulating key questions, analyzing and conceptualizing problems, defining problems and goals, discovering patterns and similarities, seeking out appropriate data, experimenting, transferring skills and strategies to new situations,

(iv) developing curiosity, confidence and open-mindedness (NCTM, 1980, pp.2-3).

One of the aims of teaching through problem solving is to encourage students to refine and build onto their own processes over a period of time as their experiences allow them to discard some ideas and become aware of further possibilities (Carpenter, 1989). As well as developing knowledge, the students are also developing an understanding of when it is appropriate to use particular strategies. Through using this approach the emphasis is on making the students more responsible for their own learning rather than letting them feel that the algorithms they use are the inventions of some external and unknown 'expert'. There is considerable importance placed on exploratory activities, observation and discovery, and trial and error. Students need to develop their own theories, test them, test the theories of others, discard them if they are not consistent, and try something else (NCTM, 1989). Students can become even more involved in problem solving by formulating and solving their own problems, or by rewriting problems in their own words in order to facilitate understanding. It is of particular importance to note that they are encouraged to discuss the processes which they are undertaking, in order to improve understanding, gain new insights into the problem and communicate their ideas (Thompson, 1985, Stacey and Groves, 1985).

CONCLUSION

It has been suggested in this chapter that there are many reasons why a problem-solving approach can contribute significantly to the outcomes of a mathematics education. Not only is it a vehicle for developing logical thinking, it can provide students with a context for learning mathematical knowledge, it can enhance transfer of skills to unfamiliar situations and it is an aesthetic form in itself. A problem-solving approach can provide a vehicle for students to construct their own ideas about mathematics and to take responsibility for their own learning. There is little doubt that the mathematics programme can be enhanced by the establishment of an environment in which students are exposed to teaching via problem solving, as opposed to more traditional models of teaching about problem solving. The challenge for teachers, at all levels, is to develop the process of mathematical thinking alongside the knowledge and to seek opportunities to present even routine mathematics tasks in problem-solving contexts.

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Chapter 2

Using The Mathematics Lesson To Teach Patience And Perseverance

Teach the children the three P's: Purity, Patience, Perseverance.

Education must award self-confidence, the courage to depend on one's own strength.

Students should not allow success or failure to ruffle their minds unduly. Courage and self-confidence must be instilled in the students.

You need determination to face the challenges of life, which is filled with ups and downs, successes and failures, joys and sorrows.

Students must develop courage, self-confidence and determination so that they can face any situation in life.

Education today does not impart to the students the capacity or grit to face the challenges of daily life.

There is over emphasis on quick and easy gains rather than patience, fortitude and hard work.

The focus in this chapter is particularly on two of the three P's: Patience and Perseverance, and will show how mathematics teaching, particularly problem solving, can be a suitable vehicle for these values. There is no doubt that mathematics presents situations which really test patience and perseverance.

Peter was a very clever eleven-year-old. In the final year of his primary schooling, there was only one test on which he scored less than 100%, and then he only lost half a mark. His classwork was always done quickly and correctly. When he knew that he could succeed, he was confident and willing to work hard. To challenge his thinking, Peter's teacher would give him some difficult problems. If Peter could not immediately see a way to solve a problem, he became a different child. He would sit, drawing on his notepad, or wander around the room. He would even ask his teacher if he could spend the time tidying the storeroom. Peter, who was normally so successful and confident, was afraid to tackle a difficult task because he was afraid that he might fail. So his solution was to quit, to make the fears go away. Fortunately, the story had a happy ending, because Peter and his teacher worked together to help him to develop more courage to tackle difficult problems rather than taking the easiest path of stopping.

Many writers have written about students such as Peter, who expect solutions to come to them quickly and easily and will give up rather than face negative emotions associated with trying the task. Another concern is that they often are not aware of when it is worthwhile to keep on exploring an idea and when it is appropriate to abandon it because it is leading in a wrong direction. They need to know when it is appropriate to use a particular approach to the task, and how to recover from making a wrong choice.

Clare, aged ten, was given the following problem to solve:

By changing six figures into zeros you can make this sum equal 1111.

$$\begin{array}{r} 111 \\ 333 \\ 555 \\ 777 \\ \underline{+999} \\ 2775 \end{array}$$

Clare selected the strategy of changing numbers in all three columns simultaneously. She worked at the task with *patience* and *fortitude* for two hours. As she worked, she said to herself, "I know that this is going to work. All I need is time, to find the right combination." After she repeated the strategy 21 times, her teacher interrupted and suggested that it might be time to look for another way to solve the problem.

In Peter's case, it was not enough for his teacher to tell him that frustration, for example, is a normal part of problem solving, and to encourage him to spend more time working on the task. Clare, on the other hand, was "overpersevering", locked into persistently pursuing one approach when it may be more appropriate when stuck to use other strategies, even such as help-seeking. One of the duties of a mathematics teacher is to help pupils to learn how to persevere when the problem-solving process becomes difficult. They also need to know how to make decisions about avoiding time being wasted on "overperseverance".

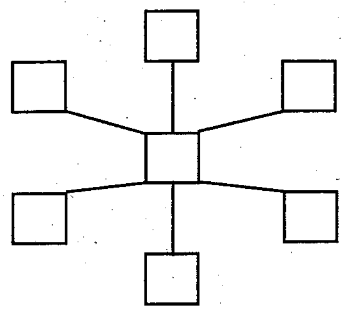
STRATEGIES FOR ENHANCING PERSEVERANCE

1. Equip learners with a range of strategies/techniques for solving different types of problems.
2. Encourage them to experience the full range of positive <u>and</u> negative emotions associated with problem solving.
3. Promote the desire to persevere.
4. Help them to make "managerial" decisions about whether to persevere with a possible solution path (when to keep trying, and when to stop).
5. Encourage them to find more than one way to approach the problem.

One sequence of strategies which is used frequently by successful, persevering problem solvers is the following:

1. Try an approach.
2. Try it 2-3 times in case using different numbers or correcting errors might work.
3. Try something <u>different</u>. (You might decide to come back to your old way later.)

This is how one student used the sequence to persevere successfully with a problem:



My goal is to put numbers in the boxes so that they add up to the same total in each direction.

These are the clues I have:
I can use each number from 1 to 7 once.
I have to find a total for each row
Row 1, row 2 and row 3 must add up to the same total.

One way I can do this is to guess. [Put numbers in randomly.]

That didn't work, so I'll try putting in some different numbers. [Repeat]

I'll try again. [Repeat]

I've tried that way three times and I don't think it is working. I'll try something different. I'll pick a total - 13 will do - and try to make the numbers add up.
2+4+7=13. I'll put 4 in the middle.
3+4+6=13
That leaves 1+4+5. That's not 13.

I'll try again. Pick 11.
1+3+7=11. Put 3 in the middle.
6+3+2=11 That leaves 4=3+5. That's not 11.

I've tried that idea twice and it doesn't seem to be working. I think I'd better try something different. I'll put 4 in the middle because that's the middle number. Then I'll match the highest and lowest numbers. The lowest is 1 and the highest is 7, so 1+4+7=12. That leaves 2 as the lowest and 6 as the highest, so 2+4+6=12.
That leaves 3 and 5, so 3+4+5=12. That's it.
I found the numbers, but I had to try it three different ways.

We can see that this student was demonstrating the values of *patience, fortitude*, and was remaining *unruffled by ups and downs, successes and failures.* The student tried an idea for long enough to give it a chance to work, but knew when it was a good time to try a different approach.

Now, let us see how Clare's teacher helped her to persevere successfully with the problem on which she had been overpersevering:

In this second attempt, Clare began by using the strategy of changing numbers at random. After four attempts I suggested that she should try a different strategy. She did not have any ideas, so I suggested that she might look at the problem column by column. She started with the units column and used the strategy "I have to get rid of 4, so change 3 and 1." She continued this strategy into the tens column, "21 - get rid of 6 so change 5 and 1" and the hundreds column, "27 - need 11 - get rid of the bigger numbers, 9 and 7".

Question 2 (with prompting):

In this magic square, each row, each column and the two long diagonals must each add to the same total and each of the numbers from 1 to 25 is used once and once only. Find the missing numbers.

		25	18	11
3	21		12	
	20	13		4
16	14			23
15		1		17

At first Clare had some problems with this question. She did not read the question before starting. She started on the right track and knew that all rows and columns had to add up to 65. She did not read that each number could only be used once. After she changed numbers three times I suggested a fresh start. Clare selected a different row, but again it was one with two gaps and she encountered the same problems. After another three attempts I intervened. We discussed the problem and Clare worked her way across the square pointing out that they all had two or more missing numbers. It was at this point that she found that one column only had one gap. Clare went on to solve the question with no further difficulty.

Question 3 (with prompting):

□x△= 91
□-△= 91
What can you put in □ and △ to make both of these true?

Clare was keen to begin this problem after the confident note on which we had finished the previous question. She began by randomly selecting 12x9, an approximate guess. She then tried verbalising multiplication facts:

10x9=90 - not high enough
11x9=99 - too high.

She then looked unsure of what strategy she should adopt. I provided a hint, that "having one on the end is hard isn't it, not many multiplication facts have a one on the end". This did not prove to be helpful so I suggested a change of approach, looking at the subtraction part of the question. This seemed to help Clare as she began to list subtraction facts beginning with 20 and giving an answer of

hb6: 20-14
19-13
...
13-7

She then said, "Now I'll go back and see if any of these multiplied equals 91". Thus she was able to select her own strategy and was successful.

<u>Question 4 (without prompting):</u>

> What is my mystery number?
> If I divide it by 3 the remainder is 1.
> If I divide it by 4 the remainder is 2.
> If I divide it by 5 the remainder is 3.
> If I divide it by 6 the remainder is 4.

In solving this problem, Clare randomly chose a number and divided it by 3, 4, 5 and 6. When this did not produce the desired answer, she tried it two more times, using different random numbers. As these attempts were both unsuccessful, she decided to change her strategy. Clare thought of using a strategy which had been suggested to her in a previous problem solving activity, that of developing a system in her choice of numbers. As the mystery number had to have a remainder of 3 if divided by 5, it had to either end in 8 or 3, as each multiple of 5 ends in either 5 or 0. Clare then wrote down all the numbers that, when divided by 6, had a remainder of 4, as this was the largest of the numbers in the clues and therefore would require going through fewer figures. Of these she looked for the numbers ending in 8 and 3 and divided them by the various numbers in the clues. On her third attempt, she found that 58 was the mystery number.

As can be seen from the above examples, Clare showed increasing *courage, self-confidence* and *independence* in her ability and willingness to *face the challenges* of the problems. At first she did not use the strategy instinctively. On problem 2, it was necessary for the teacher to intervene several times, to prompt her to change strategies and to suggest some alternative ideas. The need for teacher intervention had decreased by problem 3, and by the time she reached problem 4 she was able to recognise for herself when it was appropriate to change strategies.

The following procedure can be useful for teachers to follow, in teaching students how to persevere effectively with a task.

RECOMMENDED PROCEDURE FOR INTRODUCING PROBLEM SOLVING MANAGEMENT MODEL

1. Give the student a preliminary problem to solve without guidance. Observe whether the student instinctively used the model.
2. If the model was not instinctively used, introduce it and work through a second problem, demonstrating how to use it.
3. Ask the student to repeat the first problem while you guide him/her to use the model, i.e. prompt the student to change to a different approach after a maximum of three repetitions of the previous strategy.
4. Give two more problems, monitoring the strategy pattern and reminding students, when necessary, to follow the model.
5. Give a fifth problem and ask the student to try to follow the model, changing approach when appropriate, without any prompting from you.

<u>How Have Teachers Found this Model to be Helpful?</u>

It promotes flexibility of thinking by encouraging the students to consider alternative approaches and make decisions about when to change a particular strategy.

It discourages fixation on one particular approach and frustration which comes with this - a fresh approach not only creates new challenges, but can also lead the problem solver to a quicker path to achieving success.

It prevents a person from spending too long a time on any one possible "dead-end" strategy, yet allows the problem solver to come back to an earlier strategy if necessary.

It provides individuals with a starting point and steps to follow which stimulate more confidence to complete a problem.

It develops confidence, knowledge and ability to formulate other approaches.

It also offers a framework for students lacking confidence with which to tackle a problem, enabling them to become more empowered when problem solving.

It gives them the freedom to take risks and if they are not successful the framework allows them to change direction before it is too late.

How Teachers Introduce the Procedure

Introduce it in a small group setting, because of the importance of the teacher's intervention in the early stages.

Model the procedure in front of children so that they can see it being used widely, not only in specific problem solving exercises, but also in everyday situations.

Display the model in the classroom along with a list of general strategies.

Be familiar with the problem and the strategies/hints which are likely to be appropriate to use.

Be sensitive about when and how to interrupt or give advice, so the children do not think they have failed to achieve if the teacher has to interrupt and suggest a new approach.

Clearly, students still need to be taught the mathematical knowledge and heuristics which will enable them to select alternative strategies. However, the problem-solving model developed here enables efficient management of this knowledge by offering a framework to help them to decide when to explore and when not to explore an idea (Schoenfeld, 1985a). This type of structure is particularly important for those students who do not have an innate sense of when it is appropriate to abandon an idea and when to persevere with it. In offering such a framework, use of the model should also contribute to the management of some of the negative emotions which can inhibit effective problem-solving performance and lead to either giving up if the problem cannot be solved within ten minutes (Schoenfeld, 1985a), or "overperseverance" (Nelson-Le Gall and Scott Jones, 1983).

Strategies for Enhancing Perseverance

1. Equip learners with a range of strategies/techniques for solving different types of problems.
2. Encourage them to experience the full range of positive and negative emotions associated with troblem solving.
3. Promote the desire to persevere.
4. Help them to make "managerial" decisions about whether to persevere with a possible solution path (when to keep trying, and when to stop).
5. Encourage them to find more than one way to approach the problem.

References and Useful Reading

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A Question For Discussion With Colleagues, Or Action Research In Your Classroom

Teach your pupils how to use the procedure for improving their perseverance on a task. Monitor changes in their courage, confidence and determination to complete difficult tasks and commence new ones. Discuss their feelings with them.

Further Reading For The Student Or Researcher

INTRODUCTION

The focus in this section is particularly on two of the three P's: Patience and Perseverance, and will show how mathematics teaching, particularly problem solving, can be a perfect vehicle for these values. There is evidence that mathematics presents situations which really test patience and perseverance: "students have difficulty persisting in problem solving if their reaction is intense and negative, so they tend to quit and reduce the magnitude of the emotion" (McLeod, 1988, p.134). The implications of this can be linked to Polya's (1957) statement that an environment which denies the student the opportunity to experience the range of emotions associated with problem solving is failing to contribute to a vital aspect of mathematics education. One of these emotions is lack of courage to tackle a potentially difficult problem rather than taking the easiest path of stopping or asking for help (Wertime, 1982; McLeod, 1988). In fact, Wertime suggested that many children become reluctant problem solvers, to the extent that "they would rather enjoy the consolation purchased by despair than endure the fruitful stress of confronting the [problem-solving] process" (p.192). English (1984), too, described the "despair, frustration and panic ...[which] is probably more widespread than we realise or care to acknowledge" (p.77). This reluctance to solve problems was also mentioned by Lester (1985) and in Schoenfeld's (1985b) description of the common belief that "mathematics problems are always solved in less than 10 minutes, if they are solved at all" (p.372). It seems true that many problem solvers give up rather than face negative emotions because they expect solutions to come to them quickly and easily (Scott, 1988). Schoenfeld (1985a) also stated that many "potentially valuable approaches are abandoned before they can bear fruit" (p.98) because students are not aware of when it is worthwhile to keep on exploring an idea and when it is appropriate to abandon it because it is leading in a wrong direction. Schoenfeld indicated that the problem solver must be able to develop a structure for knowing when it is appropriate to use a particular heuristic and how to recover from making a wrong choice.

One of the duties of a mathematics teacher is to help pupils to learn how to persevere when the problem-solving process becomes difficult. It does not appear to be enough to just help students to understand that frustration, for example, is a normal part of problem solving (McLeod, 1988), and to encourage them to spend time working on the task. They also need to know how to make "[managerial] decisions about whether to persevere along a possible solution path" (McLeod, 1988, p.138). It is desirable to encourage perseverance, but on the other hand it is possible to "overpersevere", particularly if one becomes locked into persistently pursuing one approach (Mason, Burton and Stacey, 1987). "Overpersevering" is an unproductive strategy to employ when it may be more appropriate when stuck to use other strategies, such as help-seeking (Nelson-Le Gall and Scott Jones, 1983). This poses not only the question of how to enhance perseverance, but as well the question of how to avoid time being wasted on "overperseverance".

One of the difficulties faced by many children learning to solve problems is unwillingness or lack of appropriate managerial skills to persevere with a task until a solution is reached. If we are to encourage pupils to increase their *capacity/grit to face the challenges of daily life*, we need to understand three main things. The first is to understand what perseverance is through examining some definitions and some of the ways in which it has been measured. The second is why perseverance should be developed, and what the consequences might be if it is not. The third is to understand some of the factors which have been shown to influence perseverance. It should be noted that 'perseverance' is frequently used synonymously with the word 'persistence'. Consequently in this discussion the word 'persistence' will be used when referring to some of the literature in which it appeared. The first area for consideration is what perseverance is and how it has been measured.

WHAT IS PERSEVERANCE AND HOW CAN IT BE MEASURED?

Perseverance has been defined as:

> 'how long an individual will persist with the task at hand' (Battle, 1965,p.210),
>
> 'total time spent on a task before turning to an alternative activity '(Howard and Hunt, 1985, p.2),
>
> 'refusal to give up in the face of failure' (Dweck and Repucci, 1973, p.110),
>
> 'the amount of time [the pupil] is willing to engage in learning (Millman, Bieger, Klag and Pine, 1983, p.426)',
>
> 'the ability to ponder the problem even over a long period of time' (Schmalz, 1989, p.5).

Lester and Garafolo (1987) believed that perseverance consists of three dimensions. The first of these is desire to continue with a task until the correct answer is found. The second is the desire to resist the urge to accept any answer in order to finish the task. The third is the 'attitude of not giving up too quickly, of staying on task without regard for getting a correct answer' (p.9).

It is important for teachers to be able to distinguish between when a student has really decided to stop working on a task, and when the abandonment is just temporary. When using the term 'abandonment' to describe the subject's decision to stop persevering, Wertime (1982) referred to ultimate rather than temporary abandonment: '...ultimate abandonment, which engenders self-deprecating misery in the abandoner, even when the misery is mollified by a sense of relief; and ... temporary abandonment, which makes itself felt as abiding frustration'(p.196). We need to encourage pupils to understand that temporary abandonment can be a healthy phase in doing the task, but to help them to develop the courage and confidence to deal with the fear of ultimate abandonment.

Another aspect of measuring time spent on task is the extent to which the child is actually engaged with the task. Nelson-Le Gall and Scott-Jones (1983) and Nelson-Le Gall (1989) suggested that, if we believe a pupil who is spending a long time on a task is being persistent, we may be mistaken because in this situation 'persistence may be more akin to patience than to achievement striving' (Nelson-Le Gall and Scott-Jones, 1983, p.26). They believe that persistence is derived from a combination of experience with difficult tasks and observation of the attitudes and behaviour of peers, parents and teachers. They advocated that the time on task should be just one aspect of the overall strategies which students employ to overcome difficulty in completing a task. They also said, 'persistence may not be the only or the best strategy available to the child who is mastering skills' (p.2). They cautioned that 'persistence without help is one of the potential strategies for coping with task difficulty and may not be perceived as the only effective response' (p.11). The possible response strategies they suggested are giving up, waiting for an offer of assistance, asking another child for help, asking an adult for help, or persisting without help from others.

Zunich (1964) identified a number of behaviours which teachers can observe to identify a pupil who needs help to be more persistent (p.20):

(i) destructive behaviour (towards the object connected with the difficulty)
(ii) emotional response
(iii) changes in facial expression
(iv) motor manifestation (e.g. stamping foot, moving body, clenching fist, sucking thumb, waving hands, pulling on ear)
(v) no attempt (giving up almost at once or without exploring many possibilities)
(vi) rationalizing (refusing to continue, e.g. 'I don't want to do this. This is a stupid puzzle.')
(vii) attention seeking (e.g. 'Look what I did!')
(viii) seeking physical contact
(ix) seeking physical help (e.g. 'Hold this for me.') or mental help (e.g. 'What can I do now? How can I put this in?')

Having established an understanding of what perseverance is, the second area to be discussed is the question of why it is a quality which should be developed to its optimum level.

Why It Is Important To Develop Perseverance

There is evidence to suggest that perseverance needs to be developed because of its contribution to other desirable outcomes. These include its strong bearing upon success, upon learning, and upon the ability to cope with fear of failure. Each of these will be discussed in this section.

Perseverance and Success

Several studies have identified perseverance, in all aspects of life, as the trait which can distinguish between people who are successful and those of similar abilities who do not succeed at their chosen tasks. For example, perseverance emerged as the predominant factor differentiating between the least and most successful subjects in the Terman longitudinal study of the gifted (Franks and Dolan, 1982). Franks and Dolan pointed out that, although perseverance and high intelligence may be independent, the former is necessary for the successful realisation of the latter. Others, including Renzulli (1977), Haensly and Roberts (1983) and Schmalz (1989), also agreed that 'the ability to stick with a problem over a long period of time has resulted in many great human accomplishments' (Renzulli, 1977, p.18). Howard and Hunt (1985) referred to the work of Munger (1956) which indicated that perseverance positively affected achievement. Furthermore, Lajoie and Shore (1981) described studies of high school dropouts which indicated that these people were in fact superior in verbal ability, independence and intellectual orientation to some of the students who persisted and were successful.

Wertime (1979) and Butkowsky and Willows (1980) referred to the cycle in which non-persisters can become trapped. By failing to persist in the face of difficulty they are often denying themselves the opportunity to experience the success which could beget future persistence. 'When such children do succeed, it is likely to be at tasks that have lower difficulty levels. Such successes are far less reinforcing ... than those associated with greater expenditure [of persistence]' (Weiner, 1974 in Butkowsky and Willows, 1980, p.419).

A similar argument was presented by Swidler and Diener (1983) who said that one of the beneficial effects of mastery-orientation (i.e. effective perseverance) is a willingness to accept challenging situations which can ultimately lead to success. They cited the findings of Diener and Dweck (1978) that, following an experience of failure, 'mastery-oriented children showed an increase in persistence and performance and sometimes the use of more mature problem-solving strategies than they had prior to failure' (p.6).

Perseverance and Learning

There is evidence that perseverance is a factor which contributes to learning, or, as Nelson-Le Gall and Scott-Jones (1983) said, a means to the goal of learning rather than a goal in itself. Stipek (1983) believed not only that individual differences in persistence were associated with individual differences in intellectual performance, but also that the children who were the most successful learners were those who were prepared to look for and accept challenging tasks and persevere with them, rather than giving up as soon as the tasks became difficult.

Altshuler and Kassinove (1975) and Means, Means, Osborne and Elson (1973) also cited several studies, dating back to the 1920s, 1930s and 1950s, which have suggested a strong relationship between perseverance and learning. So too did Gagne and Parshall (1975), who linked the lack of persistence demonstrated by students who attributed failure to external factors (such as luck) with lack of school achievement.

Millman et. al. (1983) confirmed the findings of Carroll (1963) that, although it is possible to increase the time spent on task by offering rewards, doing so will not necessarily alter the quality or rate of learning. This is only effective if the learner is not 'already willing to persevere to the extent needed for learning' (Millman et. al., p.425). These findings indicate that it is possible to increase time on task when 'there is ample evidence to show that total time spent on learning is positively related to degree of learning' (Millman et. al., p.435).

Perseverance as a Means of Overcoming Fear of Failure

Unwillingness to persevere because of fear of failure is a factor which can distinguish between successful and unsuccessful people, particularly amongst those who are gifted and are unused to experiencing failure (Battle, 1965; Herbsberger and Wheatley, 1980; Wavrik, 1980; Howard and Hunt, 1985). As Wavrik (1980) explained, it is important for successful problem solvers to be 'basically nonjudgemental towards themselves and willing to make mistakes' (p.173). However he believed that many students, particularly gifted ones, are 'very hard and demanding on themselves...are often accustomed to being right and have a hard time allowing themselves to be wrong' (p.173). Consequently their caution limits their progress.

Battle (1965), and Herbsberger and Wheatley (1980) were others who believed that many underachievers lacked perseverance, particularly in problem solving situations, because of a fear of failing when they were unused to failure. A further reason for encouraging children to be persistent is not just to overcome the fear of failure, but for the more positive experience of intrinsic satisfaction to be derived from successful perseverance (Harter, 1975, Nation, Cooney and Gartrell, 1979). If a child can be encouraged to persevere with a difficult problem until success is achieved, then this is the first step towards creating within the child a 'desire to solve problems for the gratification inherent in discovering the correct solution' (Harter, 1975, p.186).

Factors Affecting Perseverance

The third area to be considered in this review is the exploration of factors which affect perseverance. This section will examine its relationship to other variables which have been shown to influence it. These include achievement motivation, the nature of feedback/ reinforcement, attribution (locus of control) for success and failure, the influence of role models, and confidence.

Achievement Motivation

Barling (1982) pointed out that the motivation to complete a task is a factor which should only be addressed when the problem solver has been equipped with the skills necessary to complete the task. Nevertheless, achievement motivation, that is motivation to achieve success or to avoid failure, has been the subject of much persistence-related research.

Battle (1965) found, with junior high school students, a correlation between level of expectancy for success and task persistence. Feather (1968), Smith (1969), Campbell and Rolando (1981), and Grabe and Latta (1981) also claimed that achievement motivation is related to academic persistence. In fact, Campbell and Rolando found that it was the best predictor. It may not be so much actual ability as the children's perceptions of their abilities and subsequent motivation for success which can influence persistence (Battle, 1965).

Feedback/Reinforcement

Several studies have indicated a connection between perseverance on a task and the nature and quantity of reinforcement received. In particular it has been found that partial reinforcement, or in some cases no reinforcement, leads to greater persistence than does continuous reinforcement (Chapin and Dyck, 1976; Means et. al., 1973; Millman et. al., 1983; Hamilton and Gordon, 1978; Nation and Massad, 1978). The suggested explanation for this is that continual encouragement may produce anxiety which lowers performance. In fact, Gordon et. al. (1977), basing their assumption upon the earlier work of Masters and Christy (1974) and Masters and Santrock (1976), suggested that successful perseverers may adopt a pattern of 'covert self-reinforcement' (p.1716). This possibility was also proposed by Means et. al. (1973) who said that 'persistence on some tasks (may be) lengthened when the subject develops his own reinforcement strategy and thus has internal control over the situation rather than simply responding to mechanically imposed external reinforcement' (p.9). That internal reinforcement can affect the length of time spent on a task was indicated by Tinsley (1982) and Newby and Alter (1989). In Tinsley's study of four to five and a half year olds, those who persisted longer on a cognitive task were the ones who repeated the statement, 'I must keep going until (experimenter) returns' rather than generating statements about things they liked to do.

Others have investigated the comparative effects upon perseverance of positive and negative reinforcement. There is conflicting evidence about whether negative reinforcement leads to greater perseverance than does positive. Some researchers (Nation et. al., 1979; Wyer, 1967; Chapin and Dyck, 1976) have found that it does, particularly occasional negative reinforcement after some success has been achieved on earlier tasks (Chapin and Dyck, 1976). However, there is also evidence which contradicts this theory (Masters and Santrock, 1976; Harter, 1975; Van Hecke and Tracy, 1987). Masters and Santrock worked with nursery school children, asking them to verbalise either positive or negative statements about the tasks they were given. They found that the children who made negative comments persisted for less time than those in either the control or positive conditions, and showed a striking decrease in the length of time they were prepared to persist at tasks. Those who persisted the longest were those who verbalised positive feelings. Harter indicated that positive feedback increased the perseverance of children least likely to persist alone. Van Hecke and Tracy (1987) also found that children who were encouraged persevered longer than children who worked alone. Draper (1980-1) suggested that this may be because 'telling children that [they are] "Right", "Correct", or "Doing very well" may convey to them previously unrealized information about their performance' (p.32). He also commented that negative reinforcement may have a similar effect. He suggested that praise immediately following success is more effective in encouraging perseverance than praise given regardless of success.

Another interesting finding related to the nature of feedback was that of Hamilton and Gordon (1978), that preschool teachers with the least persevering children gave, as well as more criticism and more direction, more physical comfort. This, with their other findings, suggest 'that the

lower persistent children receive more attention for dependent behaviour in the classroom' (p.465).
A further aspect of feedback which has been shown to have an effect on persistence is the internal feedback resulting from previous successes or failures (Feather, 1966; Wyer, 1967; Eisenberger and Leonard, 1979; Wertime, 1982). Wyer (1967) suggested that the difficulty of the task is a factor to be considered, as subjects persevered more on an easy task following failure than following success, and more on a difficult task following success than following failure. Her explanation of this was that if a task is easy, then subjects feel that they have little to gain from being successful but a lot to lose from failure whereas if it is difficult they feel they have little to lose from failure but much to gain from success. Eisenberger and Leonard (1979) found that initial failure leads to greater persistence, whereas Feather (1966) tended to favour initial success. This apparent contradiction may well add weight to Wyer's thesis that the effect of the prior experience could be related to the nature of the task.

Some research (Nation and Massad, 1978; Thomas and Pashley, 1982) has indicated that it is possible to increase perseverance through modifying reinforcement. Nation and Massad (1978, p.449) presented a four step model for persistence training in which subjects are placed in situations where they are likely to 'fail' and trained to face these situations with reinforcement gradually being withdrawn. Kennelly et. al. (1985) also found that an intermittent schedule of reinforcement was likely to increase persistence in the face of failure.

Thomas and Pashley (1982) also investigated a method of improving perseverance on a variety of classroom tasks. They told their subjects that 'the secret to becoming better workers was to know how to say the right things to yourself, especially when you're right in the middle of a difficult job' (p.135). The experimenter modelled and then discussed with subjects the effects of such statements as 'I'm going to try this even if it's hard' and 'I'll take it one part at a time'. They found that in the two month period of their study subjects who had received this persistence training significantly increased the amount of time they were prepared to spend on task, whereas those who did not have the training showed a decrease in persistence scores.

<u>Attribution/Locus of Control</u>

It has been suggested that it is the interaction of reinforcement and causal attribution, rather than reinforcement alone, which is a powerful contributor to perseverance. Causal attribution, or locus of control, has been the subject of a considerable amount of research. Bar-Tal et. al. (1981) defined it as falling into two categories: internal (e.g., ability and effort) and external (e.g. task difficulty and luck). They further subdivided the categories into stable (ability and task difficulty) and unstable (effort and luck).

There is consistent evidence to indicate that children, in a wide age range and a variety of tasks, who attribute success to internal factors are more persistent than children who attribute it to external factors (Gagne and Parshall, 1975; Gordon et. al., 1977; Haines et. al., 1980; Swidler and Diener, 1983). Dweck and Repucci (1973) found, with fifth-grade children, that 'those subjects who persisted in the face of prolonged failure placed more emphasis on the role of effort in determining the outcome of their behaviour' (p.109). Chapin and Dyck (1976) found that children's perseverance, in reading, could in fact be improved - not by reinforcement alone, but by a combination of this and attribution retraining, that is training subjects to attribute success to internal factors such as ability and failure to external factors such as luck or effort. Wolleat, Pedro, Becker and Fennema(1980) hypothesised that the principle of causal attribution may be related to perseverance in mathematics. They related their comments specifically to mathematics and suggested that students who attribute success to ability are more likely to persevere than if they attribute it to effort or luck. They also suggested the reverse, namely that students who attribute failure to ability will persist less than those who attribute it to luck or effort.

From the above findings it seems that perseverance should be improved by training subjects to attribute success to internal factors and failure to external. It has been shown that it is possible to affect attribution retraining (Dweck, 1975; Thomas and Pashley, 1982; Fowler and Peterson, 1981). Dweck (1975), after training subjects to take responsibility for failure and attribute it to insufficient effort, found a significant change in attribution which appeared to carry over from the specific training tasks into general classroom activity. Fowler and Peterson (1981) found, with fourth, fifth and sixth grade children performing a reading task, that direct attribution training (rehearsing and saying to themselves, 'No, I didn't get that right. That means I have to try harder.') may be more effective than indirect training (having someone else tell them to try harder). Fowler and Peterson also reported Rhodes' (1977) finding that attribution retraining can be generalised to tasks not included in the training.

Role Models

Some research shows that the perseverance of a role model can affect the perseverance of an onlooking child. Zimmerman and Ringle (1981), working with first and second grade lower class children, found a significant positive relationship between observation of a highly persistent, confident role model and persistence time on a problem. These findings were similar to those of Zimmerman and Blotner (1981), with white, middle class children, that model persistence was more useful than model success in encouraging children to persevere longer at solving a problem.

McArthur and Eisen (1976) considered the effect of gender of the role model. Working with preschool children, they read either a story depicting persistent behaviour by a male but not by a female, or one which was the reverse. They found a tendency for boys to persist longer on a task if they had been exposed to the former model, and girls to persist longer after exposure to the latter. Whereas girls exposed to the persistent male model persevered slightly more than a control group which was not given any model, boys exposed to the female model persevered slightly less than their control group counterparts.

Confidence

It has been suggested that confidence, self esteem or self perception is a factor affecting perseverance, with people high in this trait being highly persistent, irrespective of ability (Butkowsky and Willows, 1980; Wertime, 1982; Shrauger and Sorman, 1977). Van Hecke and Tracy (1987) found that expectation of success had a significant effect on perseverance time. In a study of introductory psychology students, Shrauger and Sorman found subjects who were low in self esteem persisted less following failure than did those high in self-esteem. They suggested that this may occur because people who lack confidence may believe they have little chance of mastering the task and that failing to persist means that they never have the experience of succeeding at a task which could enhance confidence. Following this argument, they found that if low-esteem subjects were placed in situations where they achieved success, or were led to believe that they were developing competence, they did improve their persistence time. Along similar lines Haines et. al. (1980) suggested that although there is little correlation between persistence and confidence in performing a task, there is some relationship between persistence and confidence in one's ability to do well on future similar tasks. This belief in one's ability to solve a problem is linked to what Wertime (1982) called courage span, that is 'courage' to have more than one attempt at a problem. He suggested that many children lack the courage to persevere with a difficult task because they believe that 'problems are do-or-die ventures at which one gets a single try, like certain games on the carnival fairway' (p.193).

From the literature reviewed it appears a matter of consensus that perseverance is a quality which should be enhanced. It has been suggested that it is possible to increase persistence time by affecting changes in variables such as achievement motivation, reinforcement, locus of control and the perseverance demonstrated by a role model. However, the interaction between perseverance and these other variables suggests that it would be a complex attribute to 'teach'.

The previous research reviewed in this chapter has been concerned with suggestions as to why some people persevere and others do not. It is also important to address the question of why some of those people who are motivated to persevere, or engage with a task for a period of time, actually spend their time successfully, whereas others who are equally motivated to persist at the task for the same length of time are not successful in solving the problem. Of particular interest is the quality of time spent on the task (Schmalz, 1989) and whether managerial strategies used by the successful 'perseverers' are different from the ones used by those who give up.

Management of Perseverance Time

One managerial strategy which may distinguish between successful and unsuccessful perseverers is knowing when and how to seek help effectively (Nelson-Le Gall and Scott-Jones, 1983, Nelson-Le Gall, 1989). There are two types of help-seeking described by Nelson-Le Gall and Scott Jones: executive help-seeking (e.g. wh.. questions) places the responsibility for solving the problem on the helper, whereas instrumental help-seeking (e.g. yes/no questions) places it on the seeker.

Not all children, however, are efficient at employing strategies such as seeking help. It is also important to be aware of those children who are willing enough to spend time on the task, but are inclined to 'over persevere', that is spend too much time working unproductively on a task rather than moving onto something else when they believe that finding a solution is not within their abilities (Swidler and Diener, 1983). Sometimes they do this because they do not want to admit to having failed (Nelson Le-Gall, 1989). Overpersevering is an ineffective strategy to employ when there may be others, such as help-seeking, which are more appropriate to use at certain times (Nelson-Le Gall and Scott-Jones, 1983). Hence the child who is often rewarded in the classroom for spending long periods of time on task may in fact be as ineffectual as the one who shows no persistence at all. Considerable support has been given to the notion that flexibility is an important component of successful problem solving (Lester, 1985; Buchanan, 1987; Mason et al., 1987; Schmalz, 1989). Mason et al. emphasised the importance of reflective thinking, particularly in planning the strategy to be used and in seeking insight or fresh ideas. Lester and Schmalz also pointed out that successful problem solvers will consider more than one strategy before attempting a solution and will have a sense of when to try different tactics. A study by Taplin (1994,1995) found that the critical factor in developing "successful" flexibility is the ability to explore different approaches with the same information rather than switching to a new set of data.

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"A society without values will cease to be human."

Chapter 3

Using The Mathematics Lesson To Create Unity, Co-Operation, And The Spirit Of Mutual Regard

Unity is vital for all.

Students should start with cultivating the spirit of mutual regard and harmony.

Unity of minds, natural love and co-operation, are the qualities we have to develop today.

By example and precept, in the classroom and the playground, the excellence of intelligent co-operation, of sacrifice for the team, of sympathy for the less gifted, of help...has to be emphasised.

The quotations above refer to the values of unity, co-operation and mutual regard which are essential to a peaceful community, whether it be in the classroom or in society in general. Current research about teaching mathematics suggests students can come to a better understanding of many mathematics topics if they have the opportunity to work together in pairs or small groups. To do this successfully, teachers need also to teach their pupils the skills of working with others in a truly co-operative situation. This chapter will suggest how this approach to teaching mathematics can help not only to enhance mathematical understanding, but also to develop character.

Teaching mathematics through co-operative groupwork can work equally well for older students as for younger ones.

Example of a Co-operative Group Activity

The International High School has thirty-nine language groups, multiple levels of English proficiency, and different mathematics achievement levels.Students often find that word problems are obstacles in learning mathematics. To help them in this area, teachers encourage students to create, write, and solve their own word problems. One technique is to have groups of students select strips of paper from paper bags labeled "characters", "theme", and "plot". Descriptive words and phrases are written on these strips. A strip from the "character" bag might say "supermarket cashier", "grocery shopper", "manager", or "butcher". The students are asked to think of possibilities within a given situation and to create a word problem. For example, they might conceive of a word problem that involves finding the price of an item after a 10 percent discount and act it out using the roles of customer and cashier....Another technique is to remove the question from word problems. This forces the students to focus on the stated relationships in the problem, predict a conclusion, and then create a question that leads to a solution. For example, the teacher might remove the last sentence from the following word problem: "The Mets lead the Giants by three runs. The Giants have two runs. *How many runs do the Mets have?* This leads to lively discussions because the students often have to defend their conclusions to their classmates.

from Santiago and Spanos (1993), p.135

Healy (1993) described a typical co-operative group discussion lesson, in which the groups discussed the following topics:

Groups 1, 3, and 6....had discovered different methods for finding the centre of a triangle. They prepared their presentations for a panel presentation to the class. The class would then decide which method was correct.
Groups 4 and 5 had asked if they could combine to continue the discussion they had initiated the day before about the fourth dimension.
Group 2 discussed this statement: "A line has to be straight and has starting and ending points."
Group 7 discussed this question: "If a monogon is a shape with only one angle, is an angle a monogon? If not, is a teardrop the only monogon?."
Group 8 found definitions for *flat, horizontal,* and *degree.* (p.241)

Healy's assessment of this type of teaching approach was as follows:

"They had to learn to work together, to rely on one another, to experiment, to discuss, to present ideas, to compromise, and to really understand the definitions and theories..." (p.233)

One of the students wrote:

"I think I changed a lot. This class changed a lot of things. We all learned to believe in ourselves and in each other.... And, you know, now I think I can handle just about anything in my life" (p.243).

Being part of a successfully functioning group can enhance self-esteem, and help pupils to develop respect for their classmates' abilities.

A Teacher's Thoughts About Co-operative Discussion

After experimenting with co-operative discussion in her classroom, Ms Cheung made the following comments:

Listening to what children say during discussion offered me a continuous and detailed means of assessing their understanding and progress. Before this session I doubted whether talk/discussion could be obtained in working with a class of thirty-six children. The class was formed into groups, which would discuss mainly on their own. I interacted with these groups by circulating. I controlled a second level of interaction between groups, by calling on spokespersons to report, and drawing in other children appropriately. I reinforce my belief that children need more opportunity to talk about their mathematics. I learnt that children working together not only have the opportunity to listen and learn from each other, but also to try out some ideas in a non-threatening environment. Every member of a group has the chance of seeing the activity in more than one way than if they were working alone.

Team work can lead to better development of mathematical understanding because of the communication that must occur for the group to function. These activities necessitate that children use all four components of language skills: speaking, listening, reading and writing. Interactions are indeed the heartbeat of the mathematics classroom. Mathematics is learned best when students are actively participating in that learning. One method of active participation is to interact with the teacher and peers about mathematics.

The social skills of co-operating with others in a group have to be deliberately taught:

each group needs to have at least one member who can explain the topic to others,

all members of the group must be responsible for the welfare of every other member,

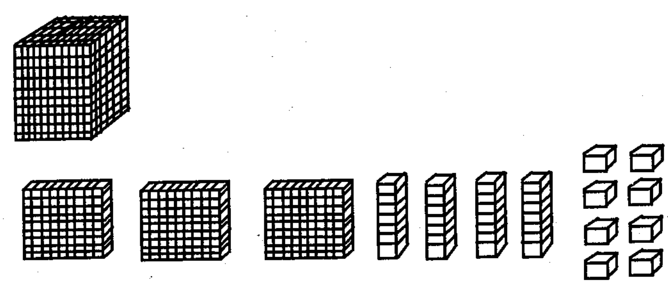
the teacher needs to demonstrate how to help another child without just giving the answer,

students need to be genuinely dependent on each other to be able to complete the task,

pupils should discuss what they did in the group.

Example of a Co-operative Group Activity

Alex, Jenny and Michael were asked to find the answer to 1000 - 348. At first they did not know what to do, because they were confused by all of the zeros. The following extract from the lesson shows how each contributed their own strengths to solve the problem.



Alex: Let's make a model. [Using Dienes (MAB) Blocks as pictured] the cube can be 1000. We need to take away 3 flats, 4 longs and 8 minis.

Jenny: Oh yes. Now I can see what we can do. We can change the cube for 10 flats, and put them in the hundreds column. Now we can take away the 3 flats, but we still can't take away any longs or minis.

Alex: We can do it again. We can take one of the flats, and change it for 10 longs. {They do this, and put the longs in the tens column.] We still can't take away the minis, so we can change a long for 10 minis, and put them in the units column.

Teacher: Michael, can you explain why they changed the cube for flats, cubes and minis? [Michael does not respond.] OK, Michael, if you could cut up the cube, how many flats would you be able to make?

Michael: [counts] 10.

Teacher: So what can we say about the cube and the flats?

Michael: One cube can be broken up into ten flats - so one thousand can be broken up into ten hundreds.

Teacher: That's right. Now, can you explain to Jenny and Alex why they broke up the flats and longs?

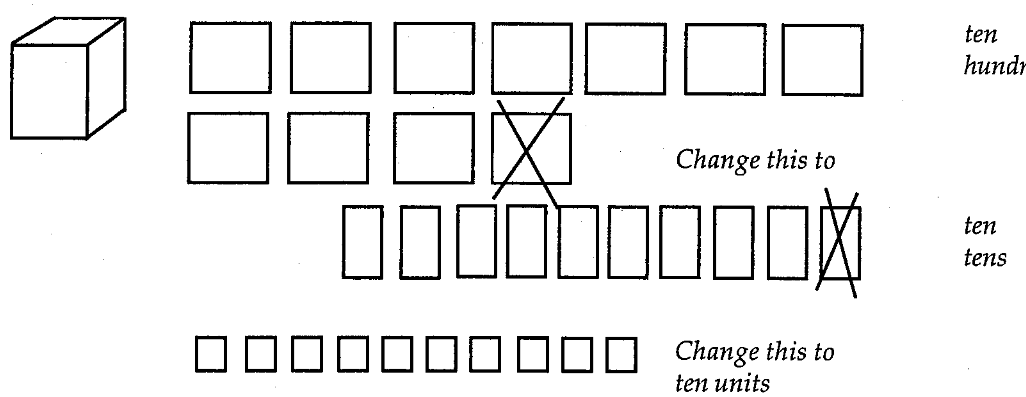
Michael: You took one of the flats and broke it up into 10 longs. Jenny: What is one of the flats worth? Michael: One hundred, so you took one of the hundreds, and broke it up into 10 tens.

Alex: Where do you put the 10 tens?

Michael: In the tens column.

Soon, with their coaching, Michael was able to explain clearly what Jenny and Alex had done. Later the teacher asked the group to find a way to record what they had done. This time it was Michael who had the idea of drawing a diagram, and using numbers and arrows to show what they had done:

1000 becomes --->



Next we took away 3 hundreds, 4 tens and 8 units

Jenny liked to look for short-cuts, so she suggested that they could leave out the diagram, and record by just using the numbers and arrows. When she explained her idea, the others agreed that it would be a good approach.

Discussion - between teacher and pupils, and between pupils themselves - is very important. The teacher can encourage the pupils to think, and to help each other, by asking questions like:

Tell me about this.
What does this mean?
How did you get that?
What did you think about next?
Can you explain...?
What did you do to get this?
What is the first/next thing you think of?
How did you figure that out?
Why is this step necessary?

Kliman and Richards (1992) suggest five steps in helping students to write, share and discuss mathematics stories (p.141).

Begin by reading a story or relating an anecdote about an everyday problem or situation in which mathematics plays a role, for example determining what possibilities and constraints a particular budget poses. Invite students to think about similar experiences they have had and to attend to ways in which they use mathematics outside of school. Then encourage them to present their own mathematical experiences to the class in a vivid and realistic manner - by telling stories; by drawing and showing pictures; and by performing skits. Let the students create their own mathematics stories. As students incorporate mathematical information and relationships into stories that are meaningful to them, they learn that mathematics can be used to model familiar situations and that it can help them make sense of the world around them. Give students opportunities to consider their own values and preferences, as well as the results of their calculations when solving problems. As students bring their everyday perspectives, experiences, and problem-solving skills to bear on the solution process, they learn that mathematics has an important role to play in everyday problem solving. Create an environment in which everyone can do mathematics. When discussion is an important part of the problem-solving process, many students who do not usually do well at school story-problem-solving tasks have a chance to succeed. Help students communicate mathematically. The teacher can help students learn appropriate mathematical inquiry and discourse (as well as such social-interaction skills as polite listening and questioning, turn taking and sensitivity to different cultural and learning styles) by serving as a "model" participant in the discussion.

A Question For Discussion With Colleagues, Or Action Research In Your Classroom

Ask your pupils to keep a check-list of the mathematics skills and social skills to be gained from doing co-operative activities. Monitor these over time, to see if there are any changes or developments. Report on these changes to your group.

Further Reading For The Student Or Researcher

Many recent research theories have advocated the use of co-operative discussion in helping learners to build up conceptual understanding of mathematics and become mathematically literate (for example, Schoenfeld, 1985, 1987; NCTM, 1989; Von Glaserfield, 1991). In fact, as Schoenfeld (1994) pointed out,

"many of the problems considered central are too big for people to solve in isolation. In consequence an increasingly large percentage of mathematical and scientific work is collaborative.... [that] requires and fosters shared perspectives" (p.58).

Co-operative group work can be used in conjunction with many models of teaching, such as constructivist, skill practice, data collection, or laboratory investigation(Davidson and Lambdin Kroll, 1991).

Models for classroom interactions have been proposed which include whole-class discussion, co-operative grouping, and pair problem solving (Cobb, Wood and, 1993). It is linked to the social constructivist notion that people learn most effectively by constructing knowledge in social settings (Vygotsky, 1962). The use of discussion is fundamental to this notion (National Research Council, 1989), and there is research evidence (for example, NCTM, 1989; Davidson and Lambdin Kroll, 1991; Webb, 1991; Cobb, Wood and Yackel, 1993; Hart, 1993) which has suggested that the appropriate use of discussion improves students' mathematical achievement.

Yackel, Cobb and Wood (1993), for example, believe that "collaborative discourse can help children clarify their own understandings by talking....and by reconceptualising their own cognitive structures as they attempt to make sense of their partner's explanations" (p.35). Not only can it enhance learning, discourse can also be useful in enabling the teacher to gain insights into whether the students have inaccurate or incorrect understanding (Wood, Cobb and Yackel, 1993).

Young children are naturally inclined to want to help each other, and this tendency can be capitalised on to encourage pupils to practise new or difficult skills (Behounek, Rosenbaum, Brown and Burcalow, 1988). Pupils can supply background information that others do not have (Hart, 1993). As well as developing skills of co-operation, having the pupils working together for some of the time can free the teacher to devote more quality time to individual or groups of pupils, rather than being needed in many different places at once. Research has shown that these strategies can be used equally as effectively for older learners.

Behounek et al. (1988) reported that co-operative grouping can help pupils to feel more accepted by their peers, and to enhance their self-esteem, as well as increasing the quality and quantity of the time they spend on task. They report that the pupils, too, respond favourably to co-operative activities (p.13):

Kate:	When I have trouble, somebody is always there to help me so that I can get done faster and do a better job on my work.
Lisa:	I like it because I feel like I am sure of what I am doing when we get together and I can ask my team if I am not sure of something.
Tracey:	I like groups because I don't know all the words and others can help me.
Jamie:	If we keep coming up to ask the teacher questions, she won't get done with her work and then she won't be able to do the really fun things with us.

As well as increasing awareness of their own abilities and the development of confidence in them, co-operative learning can encourage pupils to develop respect for their peers' abilities (Martin, 1987). "An atmosphere of mutual trust exists such that each child's opinion is respected by the others, and the teacher is necessarily sensitive to the possible potential mathematical constructions a child might make" (Wood, Cobb and Yackel, 1993, p.58).

Strategies for Implementing Co-operative Discussion

Martin (1987) described effective co-operative learning as being much more than just pupils working together in groups: "it involves the gradual development of the social skills needed for students to be able to learn from each other in a positive way....A maths problem is posed, it is the group's responsibility to solve it and make sure each member understands the solution" (p.28). As well as learning how to solve mathematical problems, pupils can learn valuable lessons in learning how to work productively together (Yackel et al., 1993). "The learning opportunities that arise from children's attempts to communicate with each other include those that arise not only as they attempt to resolve conflicts but also as they verbalize their thoughts in the course of a dialogue and as they attempt to interpret and make sense of their partner's verbalisation" (Yackel, 1993, p.35).

One important thing when trying to implement a new teaching approach is that the teacher should not be discouraged if it does not immediately work well. "Students in the upper elementary years and beyond have been socialized for many years in a school mathematics tradition. Initially, many students feel threatened in an inquiry classroom by demands that they develop their own mathematical ideas, justify those ideas, and critique the ideas of others. Teachers frequently hear, 'Just tell me how to do it." The renegotiation of social norms therefore is a much greater problem with students who have participated in school mathematics for a longer time." (Simon, 1993, p.106). Taplin and Kwok (1996) conducted an experiment in which a teacher attempted to introduce co-operative discussion activities into his classroom. They found that, for the first four months, the experiment was not very successful, because the pupils did not want to participate and there was opposition from other sources, including the teacher's colleagues and school administrators. However, the teacher persevered, and in the fifth month he, the pupils, and his colleagues all began to see benefits in using the approach and some effective learning occurred.

The social skills <u>will</u> have to be deliberately taught (Sutton, 1992). It is important for teachers to select groups which will be able to work effectively and constructively together, initially in pairs and later, as they become more accustomed to this, in groups of three or four (Behounek et al., 1988). One successful arrangement is to combine high-level thinkers with above-average and average thinkers, and low-level thinkers with average or above-average thinkers (Behounek et al., 1988). It is also important to consider the pupils' personalities and leadership skills when allocating groups. Each group should have at least one member who is capable of explaining the concepts to the others (Sutton, 1992). Behounek et al. (1988) emphasised that two rules should be established, that all members of a group contribute to the welfare of all other members, and that individual success is dependent on group success. To meet these aims, it is important to teach pupils two strategies: how to help another child without giving the answer, and how to work together toward a common goal. They suggest that this can best be done by the teacher modelling the correct behaviour, for example, "If I give Bill the answer, would this be an example of 'helping'?" (p.11). If one group member is unable to understand a topic or find a correct answer, it is the group's responsibility to explain how it should be done. They give an example of a group of seven-year-old pupils working together on an addition task (p.12):

Jennifer:	I think I've got it: six plus seven equals fourteen.
Brian:	Wait a minute, I think something's wrong. Let's use the counting chips and make sure we're right.
Ricky:	I think she's right.
Brian:	Look - here are six chips and here are seven more.
Jennifer:	Seven, eight, nine, ten, eleven, twelve - oh, thirteen.
Ricky:	I got it now. It's like doubles plus one.

The teacher's role is critical as catalyst and coach, in designing tasks and asking appropriate questions to guide the students' understanding (NCTM, 1989). Behounek et al. (1988) suggested that the following strategies can be useful for teachers who want to introduce co-operative discussion-based activities into their classrooms (p.13):

1. Teachers need to feel confident in their own classroom - management skills, because the responsibility for learning shifts from the teacher to the pupils.

2. Groups should be selected with care, and re-arranged if necessary.

3. It can be useful, particularly in the early stages of using co-operative learning, to break tasks down so that each group member has a specific task to complete. This helps to make group members more accountable to their teams, which is an important component of co-operative learning (Sutton, 1992). For example, pupils can be allocated the roles of encourager, checker, recorder, reader and time-keeper, and these roles can be rotated for different activities (Martin, 1987).

4. The teacher can move around the room, listening to the group conversations and giving feedback on strategies pupils are using to solve their problems. It can also be useful for the teacher to carry paper and pen to record notes about "significant moments in student and group developments of skills and concepts" (Martin, 1987, p.29).

The teacher has an important, and active, role to play as facilitator. Clopton (1992) stressed the importance of asking careful questions to build confidence. He suggested that asking a student to "Explain to me how you got this far" or "Explain the last couple of steps that you really understood" will help to build confidence and often enables them to resolve the difficulty without the teacher having to tell them what to do. This encourages the pupils to take responsibililty for helping themselves. Similarly, Dyas (1992) suggested that, if a teacher wants students to work through examples on the blackboard, it can often be beneficial to allow them to work with a partner or in a group, or to have already worked the example at their desk and been told that it was correct. The teacher can encourage the pupils to think by asking questions like:

> Tell me about this.
> What does this mean?
> How did you get that?
> What did you think about next?
> Can you explain...?
> What did you do to get this?
> What is the first/next thing you think of?
> How did you figure that out?
> Why is this step necessary?

Schoenfeld (1994, p.63) described his role as facilitator in the discussion process:

Firstly, I rarely *certified* results, but turned points of controversy back to the class for resolution. Second, the class was to accept little on faith. That is, "we proved it in Math 127" was not considered adequate reason to accept a statement's validity....Third, my role in class discussion would often be that of a Doubting Thomas. That is, I often asked, "Is that true? How do we know? Can you give me an example? A counterexample? A proof?", both when the students' suggestions were correct and when they were incorrect.

Lewis, Long and Mackay (1993) also advocated, like Schoenfeld, that the teacher should remain neutral when incorrect answers are given, and encourage students to react to each other's ideas.

Sutton (1992) believes it is important that co-operative activities are structured in such a way as to encourage "positive interdependence", with students being genuinely dependent on each other to be able to complete the task.

Martin (1987) suggested the use of a check-list named "What Did I Do In the Group?", which can include such skills as listening, taking turns to speak, encouraging others, asking questions, explaining ideas and checking each others' understanding. She recommended that pupils complete these and discuss them, as part of their own self-assessment as group members. This analysis of the group's use of interpersonal skills and overall function as a group is essential if the pupils are to be aware that they are developing the desired qualities (Sutton, 1992). She also recommended that, particularly in the early stages of introducing a topic, the teacher should impose fairly rigid times for completion of various aspects of the task.

One critical decision is to decide at what stage of the teaching process the use of groupwork is most effective. Martin (1987) reported two models. The first of these was a four-week program based completely on small group work. The other used group work activities for problem solving and concept understanding, followed by individual practice and application of skills. Sutton (1992) reported the use of groups of three to five students for going over homework, reviewing and studying for tests, and groups of two for understanding or reinforcing concepts.

One good tool to promote groupwork projects is the use of the computer programming language LOGO (Papert, 1980). Healy, Hoyles and Pozzi (1994) described a case study where a group of upper primary school pupils was asked to work together to draw some spoke patterns, then to find a relationship between the number of spokes and the angle between each spoke. The aim of the lesson was to help the pupils to develop their understanding of the concept of 360°. First, the pupils needed to discuss how to share the task so that each would take an equal share of the responsibility. They broke into pairs, and each pair used LOGO to write a program which would draw a different number of spokes. When this task was finished, they re-formed as a group and explored the connection between the number of spokes and the size of the connecting angle. The following discussion was reported by Healy et al. (p.40).

Paul: Something like the end number divided by... Sofia: Oh yeah 360 divided by ... [for a three-spoked figure with angles of 120]120 divided by 3...
Graham: Which is 40.
Paul: [after some more brainstorming] That ain't got nothing to do with it I don't think, we're stuck.
But Della had been given an idea which she wanted to check:
Della: Hold on, give me the calculator ... Wait a minute, wait a minute. Some of these numbers are wrong. I've got it now, I think. This times 16 equals 360.
Graham caught on immediately.
Graham: 360 is the total turn, 360 degrees, ain't it?
Della: Yeah and it works on this one. 4 times 90 is 360.
Graham: Which is the whole turn.

[The group then checked this rule with other examples] Healy et al. explained why this experience of the group working together was a positive one (p.40):

"all group members were involved and motivated in constructing the outcome together. We believe that the task design was instrumental in this, as it allowed the group to set up a synergy of mutual interdependence and autonomy. Every pupil shared responsibility for the task, by pairs interacting with each other and the computer to produce the different spoke designs. The software allowed them to use strategies appropriate to their level of understanding, and so everyone had something to bring to the whole group discussion and displayed some investment in the final outcome."

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Chapter 4

Understanding And Acknowledging Individual Differences

True education should make a person compassionate and humane.

There is too much emphasis on academic excellence at the expense of spiritual and moral excellence in our educational institutions today. Students are educated for examinations hoping to gain thereby a passport to well-paid jobs. Thus they are being trained for a living rather than for life, and this lopsided approach has resulted in a wide variety of disquieting developments, such as the persistence of racism, sexism and discrimination of various types in both public and private institutions....

The above quotations refer to two particular types of discrimination, racism and sexism. In this chapter, the focus is on some of the factors which can contribute to this type of discrimination, and suggestions for some strategies which can be used in the mathematics classroom to *promote understanding of and compassion towards people's individual differences.* The emphasis is on gender differences but, like the teacher whose comment appears below, we need to be aware that pupils in other minority groups can have similar experiences.

> Participating in a workshop about gender differences in mathematics has made me aware that there are other minority groups who can also benefit from me using the same strategies to encourage their participation. I teach in a boys' school, so I am not concerned about gender differences. However, in my classes there are many boys who do not participate actively in mathematics - some because they come from racial minority groups and might not have good enough language skills, and some just because they are quiet, shy boys. I can see ways to use these strategies to encourage these boys to participate more actively.
>
> Secondary Mathematics Teacher

The issues addressed in this chapter apply to a group of students who should and could participate in mathematics but do not. We should encourage these students to participate because, as Fennema (1993) pointed out, 'mathematics has been and continues to be a "critical filter" that successfully inhibits participation in many occupations and in career advancement and change' (p.2). A further reason for encouraging participation is, as Willis (1989) suggested, that mathematics should be more readily accessible to all students because of its 'potential to enrich our understanding of physical, social and intellectual aspects of our lives and through the pleasure and empowerment it can provide' (p.36). It is to be hoped that as teachers and pupils become more aware of practising *non-discrimination* in the mathematics classroom, these practices will also happen in other aspects of life.

It is likely that unwillingness to participate in the mathematics classroom arises from lack of *understanding and compasson,* which can often be unconscious, by teachers and other pupils. Consequently the chapter focuses on understanding some individual differences, and asks the question: how can we encourage more effective participation, by any students who are not participating fully, regardless of whether this is due to gender, race, or other differences? An exploration of girls' special needs might help to shed some light on this but the recommendations should include all students who are not participating fully.

FACTORS WHICH CAN CONTRIBUTE TO THE 'LACK OF PARTICIPATION' PROBLEM

These ideas can be grouped into the five categories suggested in *Girls Into Mathematics* (Open University, 1986):

(i) Mathematics and the Curriculum

(ii) Teaching Materials

(iii) Teaching and Learning Styles

(iv) Assessment

(v) Attitudes, Feelings and Expectations.

In the following section the outcomes of explorations by a group of primary and secondary teachers will be reported. A full report can be found in Taplin (1994). No conclusions can be drawn from the findings reported here because it was not the nature of the task to carry out rigorous empirical research. Nevertheless, some interesting questions have arisen. It is anticipated that teachers will use these ideas as springboards for ongoing action research investigations in their own contexts.

Mathematics and the Curriculum

Statements suggested by teachers

√ Timetable choices/clashes can prevent some students from selecting mathematics and related subjects even if they do want to study them.

√ Mathematics is seen as a power-based, male dominated department.

√ The nature of subject is often cold and impersonal and not conducive to encouraging girls.

√ Students often find it difficult to see the relevance of content taught in mathematics.

√ Girls often want a broader curriculum and view than focusing just on mathematics and science.

Teaching Materials

Statements suggested by teachers

√ Teaching materials are often male oriented.

√ The roles of males and females in curriculum materials tend to be gender-stereotyped.

Some teachers' experiences

> I investigated three textbooks used in mathematics and science classes in my school. In two of these there were distinct differences between the written or illustrated references to girls or boys. Generally, boys were shown more often setting or solving a problem and explaining the processes involved. Boys were almost always shown doing something or displaying a skill. There was a greater tendency for girls to be shown doing traditional 'male' activities than for boys to be filling traditionally 'female' roles. There was also a high proportion of shared tasks, where a boy and girl were working together on a problem. The third book was gender neutral in that it did used the second person rather than referring to either boys or girls.
>
> Secondary Mathematics Teacher

> I was interested to observe the way in which the boys and girls in my class used woodworking tools to develop spatial awareness concepts. Initially the boys were the most enthusiastic, while the girls tended to stand back timidly. With time and support, however, the girls developed their skills and eventually there seemed to be no difference in the gender of those choosing to do the activities.
>
> Kindergarten Teacher

Teaching and Learning Styles

Statements suggested by teachers

√ Teaching styles in mathematics may be competitive rather than co-operative, so non-competitive pupils might feel threatened and uncomfortable.

√ Girls tend to stand back and therefore not get equal access to resources or activities which are important aids to understanding.

√ Teachers praise boys and girls for different things - boys for being bright, girls for being hardworking.

√ Boys dominate due to physical strength, being noisier etc.

√ Teachers often encourage boys to think more, by asking more challenging questions, and waiting longer for them to give an answer, rather than telling them or showing them how to get the answer.

Some teachers' experiences

I was concerned about two things. One was the way I could use praise to develop self esteem. The other thing was the way I was involved in my pupils' activities. I chose these issues because I had got into the habit of teaching from the front of the room and responding to the students' answers with comments such as 'Okay', 'Good', 'Sensible'. I was also concerned that the girls were outnumbered by boys in the class and there was an underlying assumption that the boys were better than the girls, made particularly evident by a vocal group of boys. I consciously placed myself with different pupils in the classroom and moved to groups when asking or answering questions. I deliberately targetted the quieter children to encourage them to participate in group/class discussions. I developed a repertoire of responses to students' questions or answers, including, 'Good thinking strategy', or 'Can you clarify that response?' I allowed more response time, focused on permitting girls to respond following incorrect answers and followed their answers immediately by further questions. Although I only had two weeks in which to implement these initiatives, I felt sufficiently positive about the change in quality of the students' responses to warrant continuing this approach.

Primary School Teacher

I was worried that most of my mathematics lessons involved me talking too much, and my pupils participating very little. I decided to take the role of participant learner, attaching myself to a group to investigate a new topic, rather than being the person with the answers. My pupils all reacted positively to the experience and their participation improved. The girls seemed to adjust to the teacher's new role more easily than many of the boys. The children were happier in single sex groups, where peer tutoring occurred spontaneously. Girls were more supportive of their peers in groups than boys were.

Primary School Teacher

I chose to work with a group of children about whom I felt I knew very little. I realised that these children could have ability which was not being shown, so I decided to make a more concentrated effort to provide a variety of experiences and activities, to allow some 'non-performing' children to demonstrate their skills. I also recognised the need to discourage a group of 'noisy' boys from putting down the girls and their contributions. A colleague undertook a similar exercise with an older class. She was surprised that she knew the boys better as being more confident and responsive. She intends to investigate this further by asking a colleague to observe her teach to find out whether her suspicions are true that she is responding more to the boys than to the girls.

Primary School Teacher

I decided to talk to small groups of my students and ask them about their preferred learning styles. Many of these students said they preferred individual learning programmes, a few preferred to always work with a partner or small group and only two - the most confident and most active participators - indicated that they were most interested in a conventional classroom interaction between teacher and whole class. I also asked them to talk about the strategies they used when they were experiencing difficulties but did not want to ask for help. There were six common strategies which were used consistently:

- sit next to somebody who can do mathematics,
- switch off and sit quietly - nobody might notice,
- change groups - look for support,
- avoid the task - time wasting, daydreaming, creating distractions, getting equipment they might need,
- ask a peer for the answer or copy the work of somebody else,
- justify their inability to do the task by describing it as boring, inappropriate, no use, or saying they do not want to do it. I found that it was more common for the girls to use the first three of the strategies listed above and for the boys to use the last three.

Vocational Education Teacher

I chose to work with a group of children about whom I felt I knew very little. I realised that these children could have ability which was not being shown, so I decided to make a more concentrated effort to provide a variety of experiences and activities, to allow some 'non-performing' children to demonstrate their skills. I also recognised the need to discourage a group of 'noisy' boys from putting down the girls and their contributions. A colleague undertook a similar exercise with an older class. She was surprised that she knew the boys better as being more confident and responsive. She intends to investigate this further by asking a colleague to observe her teach to find out whether her suspicions are true that she is responding more to the boys than to the girls.

Primary School Teacher

Assessment

Statements suggested by teachers

Assessment procedures may not be fair to all. Some students may respond better to different types of assessment than to others.

Some teachers' experiences

I decided to try giving a series of different types of test questions to a group of eleven and twelve year-old students. The questions included word problems, multiple choice with numbers less than 100, multiple choice with large numbers and an open ended question. It appeared that girls tried to communicate their answers more fully than boys, for example by using diagrams. More boys than girls had correct answers for the multiple choice questions and the questions involving large numbers. After they had done the tests, I asked the children to comment on the questions which they liked the best and the least. The boys also tended to prefer these questions, where the girls liked the freedom of the open ended questions.

Primary School Teacher

> I surveyed two classes of twelve-year-old children and found that open-ended questions, multiple choice and practical work were all selected evenly by both boys and girls. In one of the classes the girls tended to prefer multiple choice because they knew the answer was there somewhere as a check.
>
> Primary School Teacher

Attitudes, Feelings and Expectations

Statements suggested by teachers

√ Many female teachers believe they are not any good at mathematics and this attitude is passed on to students.

√ Boys have negative perceptions of girls who do well at mathematics.

√ Traditionally it has been society's perception that it is acceptable for girls not to participate in mathematics and occupations which use it.

√ Many parents are likely to discourage or not be interested in girls going on in mathematics or to think it acceptable for them not to succeed.

√ Mathematics is perceived to be too hard - boys do it in spite of this but girls give up because of it.

√ Lack of teacher acknowledgement that some students have problems with participation means that the problems are not addressed.

√ Many students succumb to peer group pressure to not be good at mathematics.

√ High school is a time when some girls don't want to surpass skills of the boys they hope to attract.

√ Many girls do not want to appear 'different', to stand out.

Some teachers' experiences

> Four teachers gave their classes of eleven and twelve-year-olds a survey of attitudes towards mathematics (taken from Open University, 1986, p.39). Not many differences emerged, although most students said that they did not try hard enough and that this was the main reason for lack of success. One teacher found that both the boys and girls in her classes enjoyed working with their friends on mathematical challenges (girls: 70%, boys: 86%). However, there were also times when they needed to work independently.

In the same classes 58% of the girls believed that the boys in the class would not like it if the girls were more successful in mathematics; 64% of the boys disagreed with this. One question in the survey was about whether having a mathematics background would be useful in getting a job. This was given an overwhelming response by 83% of girls and 93% of boys. There did, however, seem to be some indication that the boys were more interested in mathematics and enjoyed it more than the girls. In one class 72% of the girls did not see the point of most of the mathematics that they did at school, whereas only 36% of the boys in the class made the same comment. Another question was concerned with students' confidence in their mathematical abilities. 88% of the girls and 79% of the boys were confident they would get most of their work right. However, 29% of the girls compared to 57% of boys felt that they understoood new ideas in mathematics quickly. Only 4% of the girls thought they were naturally good at mathematics and 67% felt that their success was due to hard work. 29% of the boys felt that they were naturally good at mathematics and 43% accredited their mathematical success to good teaching. On the other hand, in the same classes, 64% of boys expressed anxiety when reading or being asked a mathematics question, compared to 30% of girls. The boys, as a group, appeared to have greater self-confidence in mathematics, but also appeared to experience greater anxiety when faced with a mathematical task. Both girls (92%) and boys (72%) believed that if they worked carefully, they would be successful in mathematics. The girls tended to attribute success to hard work (67%) or good teaching (29%), with none attributing their success to good luck. The boys attributed success in mathematics to natural ability (29%), good teaching (43%), or hard work (21%). The girls did not attribute failure to lack of ability and did not appear to fear mathematics. They saw this as a reflection that the work was too hard (38%), lack of hard work (46%), or lack of luck (16%). In addition to attributing failure to bad luck (29%) and lack of hard work (46%), the boys also identified that failure occurs when the work is too hard (28%).

RECOMMENDATIONS

The teachers who wrote the comments shown above were asked to recommend ideas which they could try in their classrooms to encourage more understanding of those students from minority groups who may not be participating as fully as they could or should be. Recommendations include:

- Continue encouragement, mainly verbally. Value everybody's responses and have firm rules about interruptions and 'put downs'.
- Emphasise the relevance of the work to future careers or further studies.
- Provide role models of minority group students participating actively in studying, teaching or working with mathematics.
- Encourage a balance between co-operative and competitive teaching and learning styles. The former can be achieved through, for example, providing suitable concrete materials, 'open-ended' questions and an organisational structure which enhances group work. Make lessons more exciting through more practical work.
- Try to balance references in teaching materials to males and females and male/female or other groups' interests.
- Demonstrate an 'expectation' for students to participate.
- Encourage group work and peer tutoring, particularly on activity based and problem solving tasks.
- Allow students sufficient time to complete their work.
- Encourage different strategies for approaching and solving problems.

- Place mathematics in a realistic context.
- Provide a 'sympathetic' environment in which students feel comfortable to contribute responses to questions and all efforts are valued by all students.
- Enable all students to see a reason for the mathematics they are doing.
- Talk to the non-participators about the reasons for lack of participation - perhaps our perceptions are invalid.

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Some Questions For Discussion With Colleagues, Or Action Research In Your Classroom

What are the effects of making mathematics content more relevant to the <u>students</u> rather than to 'adult real life'?

Do girls perform better on a task set in a 'female' context than on the same task set in a 'male' context? Discuss with your pupils the value of setting the same task in different contexts to suit different experiences and interests.

Would those students who do not participate fully in mathematics be encouraged to do so if they were grouped together in the one class without the presence of the more assertive students?

Do different groups of students respond differently to the same topic presented in a co-operative or competitive style?

What are the outcomes of assessing the same mathematics concepts in a variety of ways?

How do different groups of students' change in their confidence to do mathematics as they become older?

Are girls more likely to participate actively when they are interested in the topic? Is it less necessary for boys to be interested in the topic to participate?

Consciously ask all pupils questions which will encourage them to think, and be patient in waiting for or coaching them to give responses.

FURTHER READING FOR THE STUDENT OR RESEARCHER

Heid and Jump (1993) stated that:

> Underutilizing America's female minority, and physically handicapped population wastes precious human resources. All students must learn to value mathematics and science and become confident in their mathematical and scientific ability. A better understanding of special populations and the factors related to their achievement in mathematics and science are required to improve the chances for female, minority, and physically handicapped students' academic and career success (p.163).

They also talked about the importance of starting intervention at an early age, and continuing it throughout the school years:

> An overwhelming number of studies indicate that the early adolescent years of 9-13 are critical for psychological, social and cognitive development in the fields of mathematics and science. It is during this stage of growth that "goal embedding" takes place... peer pressure to conform to traditional sex roles is most intense; children's opinions of themselves and confidence in their abilities are lowered or raised (e.g. female, minority and physically handicapped students' opinions and confidence change and become lower); and attitudes toward, achievement in, and aspirations in, mathematics and science sharply decline (p.161).

As in the first section of this chapter, the focus here will be on the problems of underparticipation in mathematics by girls. However, it is important for the reader to remember that most of these issues are equally applicable to other under-represented groups.

This section will link the comments made earlier in this chapter to previous research in the area of gender issues. The particular problem addressed in the chapter is that girls are often under-represented in higher level mathematics courses, choosing to opt out when the option is given to them. This focus was chosen because it reflects recent changes in thinking about the nature of the 'gender' problem in mathematics. It is no longer widely accepted that girls *cannot* perform as well as boys, nor that they *do* not but that they *will* not (Willis, 1989). Willis suggested that the girls who do participate in mathematics can perform as well as boys, although there is some evidence of males achieving better than females amongst above average students (Leder, 1993). Willis advocated that the major issue of concern is the lack of participation by girls in mathematics and mathematics-related courses, and the lack of understanding by teachers, parents and other pupils which can contribute to this lack of participation. This problem is not, of course, exclusive to girls. The issues addressed in this chapter apply to a wider group of students who should and could participate in mathematics but do not. That group of students does tend more frequently to consist of girls, but there are also many boys who fit into this category and whose participation should be encouraged in the ways described here. We should encourage non-participators to participate because, as Fennema (1993) pointed out, 'mathematics has been and continues to be a "critical filter" that successfully inhibits participation in many occupations and in career advancement and change' (p.2). A further reason for encouraging participation is, as Willis (1989) suggested, that mathematics should be more readily accessible to all students because of its 'potential to enrich our understanding of physical, social and intellectual aspects of our lives and through the pleasure and empowerment it can provide (p.36).

The ideas discussed here will be grouped into the same five categories discussed in the early part of the chapter:

(i) Mathematics and the Curriculum

(ii) Teaching Materials

(iii) Teaching and Learning Styles

(iv) Assessment

(v) Attitudes, Feelings and Expectations.

Mathematics and the Curriculum

Previous research confirms the suggestions made by the teachers whose comments appeared earlier in the chapter, that one reason why girls may have traditionally been precluded from selecting mathematics options was because timetabling constraints made it necessary for them to choose between mathematics-related subjects and traditionally 'female' subjects such as languages which they also often wish to study (Open University, 1986). This is further compounded by the suggestion that mathematically able females are more likely than mathematically able males to have a wider range of talents and to consequently have a wider range of subject options from which to choose (Willis, 1989). It is also suggested that girls may be excluded from participating in mathematics to the same extent as boys because they do not have the same opportunities to put their mathematical skills into practice in other mathematics-related subjects such as physics (Open University, 1986). Another problem lies with the content of the mathematics which is taught. This often seems to lack interest and be boring for girls (Open University, 1986). Many teachers believe that if they choose a subject of interest to the boys, the girls will also work on that subject whereas if they choose a subject of interest to the girls the boys might lose interest and become bored (Clarricoates, 1978). It is not, however, recommended that the nature of the mathematics should be 'devalued' to make it more appropriate for girls. Willis (1989) suggested that while the curriculum should make provision for the learning styles and interests of all students, it is also important for the curriculum to be 'rigorous and intellectually demanding' (p.38), which would empower all students to use mathematics.

Teaching Materials

The teacher who investigated the textbook material found outcomes which were consistent with other research. Surveys have shown that the representation of girls in mathematics and science textbooks decreases as the age of the target group increases (Open University, 1986). Also, males are more likely to appear as identifying, setting and solving problems, being more competitive and skilful, teaching mathematics skills to others and displaying initiative and inventiveness (Open University, 1986). Girls have been shown to be more likely to appear as efficient record keepers, practising and modifying already learned skills, developing themes suggested by others and setting standards of behaviours (Open University, 1986). There is a little evidence to suggest that females perform better on female-oriented tasks than on the same tasks presented in male contexts (Open University, 1986).

It was encouraging to see that the girls in the woodworking group described in the first section of the chapter increased their confidence and participation in the activity because there is some evidence that young children who play primarily with 'boys' toys show stronger visual-spatial problem solving ability than children who play primarily with 'girls' toys' like dolls, housekeeping materials and fine motor activities (Cockcroft, 1982).

Teaching and Learning Styles

It would be valuable to spend more time exploring the preferred learning styles of students who are typically 'non-participators' in mathematics, because, as Koehler (1993) suggested, the teacher's actions and words have a significant influence on the students' mathematics learning. There is a great deal of evidence to suggest that students tend to react differently to different types of questions. For example, girls tend to volunteer to answer questions requiring a 'yes/no' response and boys to answer questions requiring an explanatory answer (Cockcroft, 1982; Hall, 1982). 'Teachers need to address higher-cognitive-level questions to females as often as to males' (Koehler, 1993, p.145). There is also evidence to support the suggestion that boys are more assertive in class (Smith, 1983), for example being more likely to call out an answer, and that it is the more assertive 'participators' who receive the most assistance and encouragement (Becker, 1981; Evans, 1982). Not only do these students receive different amounts of attention, they also attract different types of attention (Fennema, 1993; Koehler, 1993). For example, there is evidence of a tendency for teachers to wait longer for males than for females to answer a question, to 'coach' male pupils more than females in working towards a fuller answer by probing for additional elaboration of explanation, and to give boys specific instruction but show girls or do it for them. Leder (1993a) reported that girls were likely to be given more of the teacher's time on routine, low-cognitive-level questions and boys were given more on high-cognitive-level questions. Koehler (1993) suggested that 'teachers might also be encouraged to respond to requests for help with hints rather than complete solutions' (p.145). What are the implications to mathematics success of helping the assertive/confident pupils in a different way from the more passive ones? It may subtly communicate that the less assertive pupils, more frequently girls, are not expected to be able to do it. Leder (1993) indicated that 'there is considerable evidence that females, on average, are reinforced and encouraged less than males to work independently and persistently, especially in difficult high-level mathematics tasks' (pp.20-21). Cockcroft (1982) said that 'in such circumstances, girls are likely to receive the message that they are not expected to perform as well as boys and to react accordingly' (p.63). Koehler (1993) suggested that the kind of help given is significant and that performance could be enhanced if teachers 'respond to requests for help with hints rather than complete solutions' (p.145). She also suggested that limiting the amount of help might contribute to students becoming more autonomous mathematical problem solvers. There is a danger that in setting people low standards they are being encouraged to conform to them and that in encouraging them to ask for too much help they may become dependent on others to solve their problems for them.

Another area where we can expect the more assertive people to be more successful is concerned with competition. Mathematics is regarded as being more conducive to competitive styles of teaching, whereas subjects such as English are generally seen as providing more opportunity for co-operative interaction (Shelley, 1982). It has been suggested that boys are more likely to choose to engage in high-level mathematical tasks and to work independently on them (Meyer and Koehler, 1993, p.69). The dilemma is whether to encourage girls to study mathematics in co-operative settings, or to develop the independent skills which higher level mathematics seems to need (Fennema, 1980). Fennema (1993) suggested that 'it could be that the most effective teaching for males is different from that for females' (pp.5-6). While discussing the need to take this into account, she suggests that 'perhaps the most important thing that a teacher can do is to expect females to work independently, encouraging them to engage in independent learning behavior and praising them for participating in and performing well on high-cognitive-level mathematics tasks' (p.6).

One of the teachers commented that his students preferred to work in single sex groupings. Leder (1993b) reported findings of others that such groupings have contributed to improved attitudes of mathematics and to more girls deciding to continue with mathematical studies, but that single sex grouping has not contributed as much to improving achievement.

Assessment

The differences reported by the teachers earlier in the chapter reflect the differences in wider contexts that boys tend to perform better than girls on objective tests (Murphy, 1978) and multiple choice questions (Cookson, 1980; Harding, 1979) and girls to perform better on essay-type questions (Harding, 1979). It has been suggested that girls might need the opportunity to be assessed using questions which give them the opportunity to draw on their verbal skills (Murphy, 1979). Murphy suggested that 'mathematics examinations may resemble objective tests in tending to test convergent rather than divergent thinking, in a way which other more open-ended examinations do not' (p.5).

Attitudes, Feelings and Expectations

There is some evidence of a relationship between attitudes and mathematical performance (Fennema and Sherman, 1976) and also evidence that group discussions aimed at improving attitudes can contribute to improvement in performance (Murphy, 1979).

Many of the outcomes reported earlier by the teachers are consistent with previous research findings. For example, there is evidence that at primary level there are not many differences between boys' and girls' liking for mathematics and their perception of its usefulness, although there is a little evidence of boys liking it more than girls do (APU, 1980; Weiner, 1980). At secondary level there are reports of girls tending to neither like mathematics as much as boys do nor to see its usefulness to their lives (Leder, 1972; 1993a; Keeves, 1973; Mitzman, 1977; APU, 1981; Russel, 1983). It is interesting to note that in the outcomes reported here, both boys and girls regarded mathematics as being important to their future careers, whereas earlier research suggested that boys regarded it as more important than girls did (APU, 1981). It was also reported (APU, 1981) that interest in a topic did not have any relationship to whether boys performed better on it, but girls were more likely to perform better on the topics they found interesting. This occurred regardless of how difficult the topics were.

The teachers commented on a gender difference in confidence, with boys appearing more confident and girls more anxious about mathematics. This, too, is consistent with other research evidence that boys are more confident than girls (APU, 1982; Walden and Walkerdine, 1982; Isaacson, 1982; Russel, 1983; Fennema, 1993), more likely to think that they can understand a new idea more quickly and be more correct in their work, whereas girls are more likely to be surprised when they succeed. These differences can occur as early as the age of 11 (APU, 1981). Eynard and Walkerdine (1981) reported that the girls who were the most successful in mathematics were the most confident and assertive. There is evidence to suggest that this difference becomes more marked at secondary school, being further complicated by the fact that many girls also become anxious about succeeding in mathematics (Leder 1980; 1993). Because of their desire for peer acceptance it becomes fashionable not to be able to do mathematics. The question of improving confidence, and particularly the concerns expressed by participants about its decrease over the years, is significant. There is evidence that confidence in learning mathematics is more highly correlated with achievement than any other affective variable (Fennema, 1979; Myer and Koehler, 1993) - moreso for females than for males (Leder, 1993).

Studies (Cockcroft, 1982; Dweck and Bush, 1976; Kloosterman, 1993) have reported that boys are more likely to relate success to ability and girls to hard work or luck and for boys to attribute failure to bad luck or lack of work and girls to lack of ability. Fennema (1993a) reported that this was also reflected in the perceptions of a group of teachers and suggests that the teachers who believe it to be true might well reinforce the attitude in their students.

Another very important issue is the attitudes of parents. There is evidence to suggest that parents' expectations do have an effect on children's participation in mathematics and are more likely to have higher aspirations for their sons than for their daughters (Willis, 1989). For example, Stamp (1979) found that both boys and girls were more likely to choose to study mathematics if their parents liked it and were good at it. In Stamp's study the girls' attitudes in particular were influenced by their mothers' attitudes towards mathematics.

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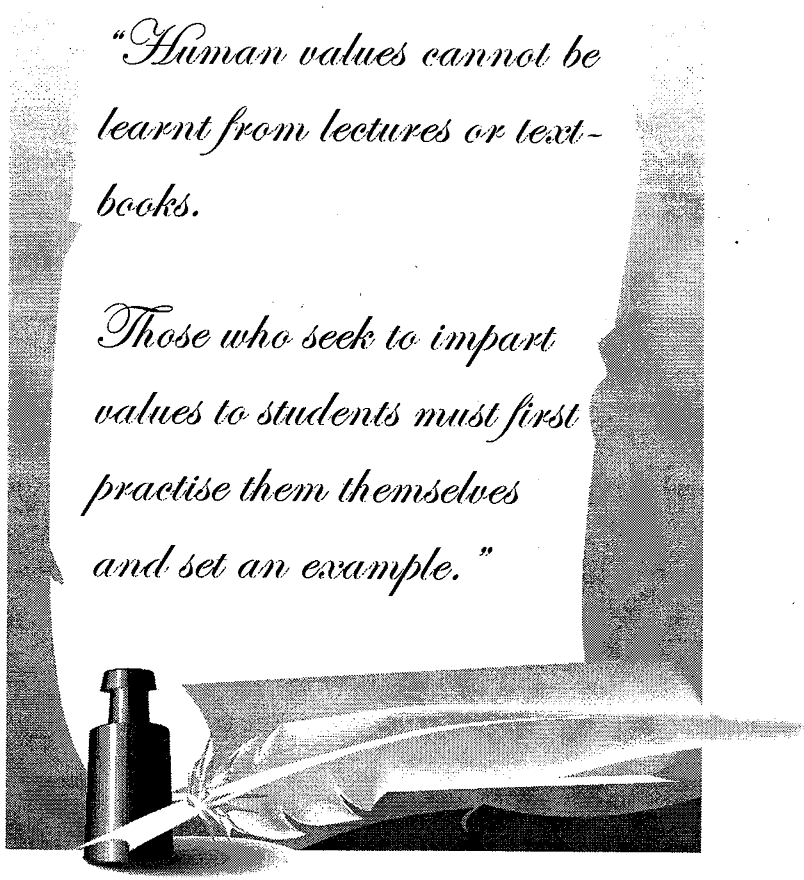

SECTION 2

Using Mathematics As A Tool To Practise Human Values

Encouraging Pupils To Be More Careful

If a student gets 25 per cent or 30 per cent marks in a subject, he is supposed to have obtained pass marks and is promoted. This only means that everyone has the licence to commit errors to the extent of 70 per cent to 75 per cent. If one commits mistakes to the extent of 75 per cent as a student, how much more mistakes will he not commit when taking up a job? He may commit even 90 per cent mistakes and get away with it. This is not a satisfactory state of affairs. "Look up and aim high" should be the motto. Low aim is actually a crime! If a student aims at 90 per cent, he may manage to get 60 per cent, if, on the other hand, he aims at 20 per cent, he may get only 15 per cent.

McAcy (1993) suggested a useful strategy for encouraging pupils to be more careful about eliminating careless mistakes.

> The strategy is to ask the students to keep a check list of the most commonly made mistakes, such as "added wrongly", "dropped the negative", "copied wrongly", "did not read the question". I use the analogy that if one always trips when walking from a tiled floor to a carpeted floor, one soon learns to be extra careful. If students are aware that they often forget to distribute a negative one, then every time they see a minus sign preceding parentheses, they should be extra careful. Each time the students do a test, they record how their mistakes were made, and look for patterns in the types of errors they make over time. A similar approach can be used for identifying weaknesses in homework assignments. McAcy found that using check-lists led to fewer students failing, more achieving As, and an increased level of confidence.

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Education must award self-confidence, the courage to depend on one's own strength.

Mathematics And Social Issues

Love of the country is turning into indifference. We have to develop in the students deep love for the country. Students should be taught how to use in a worthy and ideal manner their talents and abilities.

True knowledge is that which establishes harmony and synthesis between science on the one hand and spirituality and ethics on the other.

Education is not merely the gathering of scientific knowledge....It should instruct man to decide on what has to be done and how. It must make man recognise the kinship that exists between himself and others. The human personality must blossom into enthusiasm for work, into eagerness to raise society to the highest level.

The students do not pay the least attention to the promotion of the welfare of the society, and they deny their duty to others. They have no understanding of their social obligations. Unless the desire to do service is earnest and the skill to do service is cultivated, students will remain a burden on the community, behaving as parasites and exploiters.

Learning to earn a living is only half the job. The other half is to make life worthwhile and meaningful.

Students must develop extensive interests. They must visualise wide horizons.

True education should make a person compassionate and humane. It should not make him self-centred and narrow-minded, Spontaneous sympathy and a regard for all beings should flow from the heart of one who is properly educated. He should be keen to serve society rather than be preoccupied with his own acquisitive aspirations.

Mathematics can be used as a tool to explore social issues. The following examples of activities will show how we can encourage pupils to use mathematics to develop an understanding of their *social obligations* , ways in which they can *conserve and protect the environment* and contribute towards *the welfare of others in society.* The examples will also illustrate ways in which this can be done through the regular topics in the mathematics curriculum, without having to add any extra topics to the syllabus. The activities will enable pupils to understand the mathematical theory behind some of the issues which are considered important in today's society.

Example of Mathematics Used as a Tool to Understand a Social Issue

One of the issues of concern in today's society is gambling. There have been many cases where lives have been ruined and families broken up because of this vice. Take, for example, the lotto games. Some people believe that if they buy a large number of tickets, they will have a high chance of winning, and waste huge amounts of money because of this theory.
The following lesson plan has been taken from Lovitt and Clarke (1992, Volume 1, pp. 111-116). The topic is Probability Theory.

Maths And Lotto
When will your numbers come up?

For many people, picking six numbers out of 40 or out of 45 doesn't seem too difficult - that's only about one seventh of the numbers. But as with all gambling games, the easier it looks, the more people are convinced they can win, the bigger profit the organisers make. Do people have realistic expectations about their chances? Should they? In this activity pupils play a simplified form of lotto, work out the odds, and discover that there is often a big difference between perception and reality.

This activity involves pupils playing a simplified form of Lotto - two from six. They then analyse their results which often turn out to be rather different from their initial expectations. When this is extrapolated to the real game, pupils begin to understand the overwhelming odds against them.

1) Discuss what children know about Lotto and their expectations:
   Who knows what lotto is?
   How do you play it?
   How much does it cost to invest?
   If you bought a whole card ticket every week of your life,
   how often do you think you might win?

2) Our simplified Lotto game - two out of six
   I want to show you some of the mathematics behind lotto.
   The real game is quite complex, so let's play a very simple form of lotto which is much easier to win.
   You choose two of the six numbers.

   In your books write Game 1 and the numbers from 1 to 6 and circle your choices.
   I'll choose 2 and 6.

3) How many winners would expect?
   What do your eyes tell you?
   There are six numbers in the barrel.
   If your two numbers are drawn out, then you win.
   If we play this game many times, sometimes you would win,
   other times you would lose.
   If we were to play 100 times, write down how many times you would expect to win.

4) The Draw
First game - the winning numbers are one and six
Write down the numbers
How many winners?
Only One? Write it down.

5) Play 10 games
Play 10 games in all because we need the data for our analysis.

6) The Analysis
Just before starting this section, a good idea is to reward all the non-winners as well.

I hope you have enjoyed the game, but our reason for playing is to learn

something from the maths involved.
We have a lot of interesting information, so let's what it tells us.

My first question is about luck.
We have 26 players and we've played ten games - 260 in all.
We had 19 winners.
Do you think we were unlucky, that is, should we have had more winners?
Or were we lucky - that is, we did better that expected?
Or are the results about right?

7) The mathematics
The following analysis is done informally, appealing more to pupils' intuitions than to formal probability.
From first principles, work out all the possible pairs.

We need to know the chances of winning one game.
You win if your pair of numbers is drawn.
So how many pairs could you draw out.

We can answer this question - I'd like to show you how mathematics can assist.

8) Discussion - your real chances of winning
Since there are 15 pairs your chances are ...? (Answer - one in 15)

So what's the answer to the question 'when will your lucky numbers come up?' (Once in every 15 games)

We have 26 players - what should we expect?
More than one winner per game, but less than two.
How many groups of 15 in 26? (1.733...) So we expected on average about 1.7 winners per game.
How many winners should we have expected? (17.3)
We actually had 19, so our maths tells us we were slightly on the lucky side but very close to what we expected.

It looks like a game of luck, but the organisers have a rather good idea of what will happen.
19 winners is not surprising.

Had we only three winners I'm sure you would have been so surprised as to be downright suspicious.
If we had 150 winners then I would have been suspicious.
Actually there is a whole branch of mathematics devoted to knowing how suspicious you should be as the results get further from what you expect.

9) Comparison with initial guesses

The conclusion is the realisation that there is a lot more pairs than they thought there would be! If people are over-optimistic in this simple game, how realistic would their understanding of the 45:6 or the 40:6 game be?

10) Comparison with the real six-from 40 (or 45 game)

The following is an attempt to present these statistics in the same way as 6 : 2 game was developed and analysed.

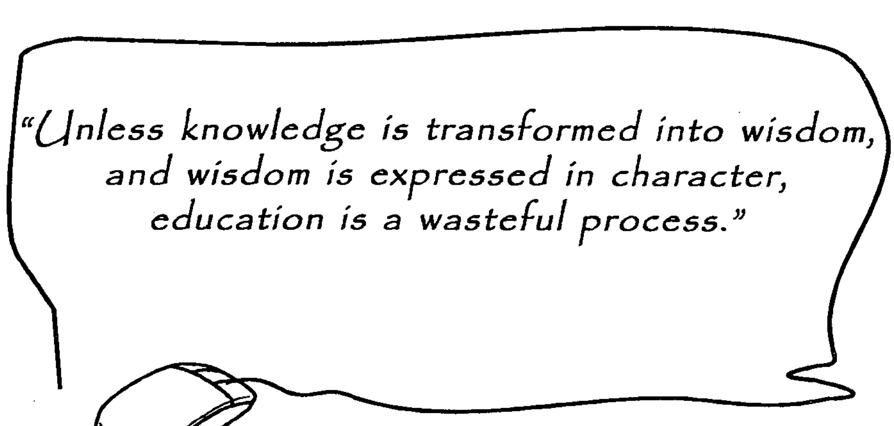
Let's look at Lotto where you select six from 45.

You win first division if your six numbers are drawn from the barrel.
So, how many groups of six are there in 45?
You could do it like this, just like you did for the 6 : 2 game.

You can see that there would be a very large number of combinations.
It's actually 8 145 060 different groups of six.
That means your chances are one out of eight million for a single game.

In reality, many people fill out a ticket with ten games which costs $2.65 (price in 1987).
So you can expect to win once in every 800 000 weeks, that is, once in every 15 000 years.

Lotto operates in several different states and attracts about $8 000 000 each Saturday.
That represents 32 million games at 25 cents a game.
Since we expect one winner, each 8 million games we expect four first division winners each week.
Keep a check on how many winners there are each week.
See how near it is to our mathematical predictions.



Mathematics And Conserving The Environment

If man is to live in harmony with his surroundings he must learn not to violate the laws of nature. He must learn to conserve and protect his environment. He must respect all life, trees, plants and animals; otherwise he may upset the delicate balance of nature, resulting in a calamity for the human race. At the same time, the world has limited resources, so we must learn to conserve whatever we have.

The following ideas have been taken from *Arithmetic Teacher* 41(1), September 1993, 27-29.

Mathematics Topic: Graphing
Graphing Trash Material

Students save or keep a record of materials they throw away. Sort these materials into categories (e.g. paper, plastic, metal). Construct graphs of the numbers of items in each category. Discuss some of the items that could be reused or recycled.

Mathematics Topics: Statistics, Graphing
Classroom Paper

Ask students to predict the number of pieces of paper they will use during the day. Keep a tally of the actual number used. Use graphs to record the numbers used over a week. Discuss the findings. Were they surprising? Do pupils think they are using too much paper? What are some ideas for saving paper? Implement some of these ideas, and collect further data for comparison. Have students examine and compare their graphs and discuss things they notice. Why might numbers vary from student to student? Why might more paper be used on some days than others? Has the campaign to reduce the use of paper been successful?

Mathematics Topics: Statistics, Graphing
Aluminium Cans

Conduct a survey amongst friends and family members to find out how many aluminium cans they use in a typical day. Use this information to calculate the average individual's can use in a year. Use the results as a basis for discussion about conservation of aluminium.

Mathematics Topics: Statistics, Graphing
How to Bag It?

Carry out a similar survey to the one described above, to find out what the numbers and kinds of shopping bags people use. Discuss the pros and cons of using different types of bags. For example, a fifteen-year-old tree is required to make approximately 700 grocery bags. How long will these bags last in a supermarket?

Mathematics Topic: Problem Solving

Imagine what would happen if a gradual reduction of material wants were practised by every individual instead of increasing his desires. The economies of whole nations would change and become more manageable, instead of veering out of control as at present. The ecology will be restored instead of the wanton destruction now taking place.

With your guidance, ask your pupils to suggest a "real world" investigation to monitor the effects of cutting back on certain desires for a week. What are the longer term effects, if they continue for a month, a year, or a decade?

Mathematics Topic: Percentage, Practice of Arithmetic Skills
Managing Money as a Resource

"It is evident that many families ... have problems with debt and figures for bankruptcies and house repossessions have soared. In part this might be caused by schools previously ignoring money management as a topic." (Duffell, 1993, p.9).

Duffell (1993, p.10) suggested the following ideas for teaching about managing money:

For the lower primary school: teaching children about the value of money:

> One school turned their hall into a supermarket for a day. A local store provided stock on sale or return and the children acted as shop keepers and cashiers. Whilst they needed a little supervision their mental arithmetic improved as they calculated four apples at 20p each and then had to work out the change from a ..5 note. Each child gained confidence [and learned about the value of money] by successfully taking on adult roles.

For the upper primary school:

> A teacher of year 5 gave lessons about pocket money and spending which helped bring arithmetic to life and gave the children a budget planning sheet which would be useful in later years when the figures became larger.
>
> In several schools the children designed a barter value list and exchanged services using bartering rather than money.
>
> A very ambitious project in one junior school....involved the school developing their own credit card for the tuck shop.

All of the above activities would be successful in helping children to understand and appreciate the value of money and to begin to develop good habits of careful budgeting, judicious use of credit cards, and alternative ways of exchanging goods and services.

For the secondary school: to teach skills of personal money management and develop mathematical skills:

Draw a bar chart showing how $1000 grows with compund interest at 14% over 40 years. What is the final balance?

Draw a pie chart showing your expected spending pattern on leaving school

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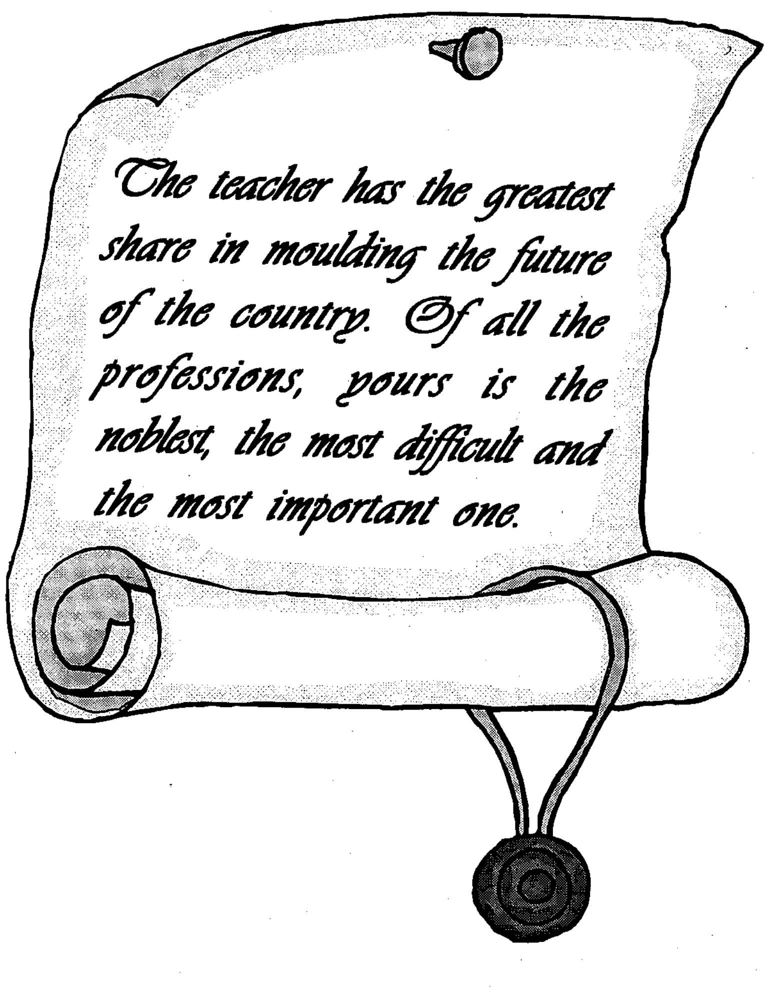


Using Mathematics To Help Others

Education does not mean imparting of verbal knowledge. The knowledge that is gathered in schools and colleges should be capable of being used for service to society and helping to improve the conditions of one's fellowmen.

Each student must, after intelligent inquiry, decide for his guidance during every occasion which goal is best and which action is best suited to realise it. Both the goal and the action must serve the needs of society and help it to progress.

Education should be utilised for developing the power of discrimination between right and wrong, good and evil.

Share your knowledge and it will grow more and more.

Real education should enable one to utilise the knowledge one has acquired to meet the challenges of life and to make all human beings happy as far as possible.

Mathematics Topics: Measurement, Estimation, Ratio, Proportion, Division, Fractions
Mathematics and Sharing Food

Help those who are in a bad position and serve those who need your help.

Sharing with others, serving others, this is the main thread of Vidya [the acquisition of real Education]. Education is rendered noble when the spirit of service is inculcated.

Children, particularly in the lower grades, spend a lot of time measuring and comparing: weight/mass, capacity, volume etc. An ideal way to practise these skills is cooking: giving them quantities of ingredients to measure, and producing a final product which can be prepared easily in the classroom. They can be encouraged to use estimation strategies to approximate the quantities, and know when "near enough is good enough" and when they have to be accurate. They should always be encouraged to develop strategies for estimation, and to discuss these with the teacher and with each other. The results can be shared with the less fortunate. Doubling or tripling the quantity of a given recipe can give practice in the use of ratio and proportion. Children can learn the process of division, and fraction concepts through dividing and sharing food equally. The challenge can be increased by asking them to share three sausages between five people. ... Please remember, though, that there is only minimal mathematical value in doing these activities unless the teacher has a particular skill in mind, and constantly reinforces this skill by careful questioning, and discussing with the pupils the mathematics skills which were involved in the activity.
"Traditionally in mathematics classrooms, the relevance of culture has been strangely absent from the content and instruction. The result is that many students and teachers unquestioningly

Mathematics Across The Curriculum To Understand Humanity Better And Understand Their Heritage / Culture

Teachers can explain how in this world we are all inter-dependent on each other. We should develop a sense of gratitude towards others who work to provide food.

During the process [of education] the Vidya [the acquisition of real Education] also instructs incidentally about the ideal configuration of society, the most desirable affiliations between man and man, the most beneficial relations between peoples, races, nations and communities, and the best mode and manner for day-to-day life.

However learned one is in worldly knowledge unless one's mind is cultured, the learning is mere junk. The system of education which teaches culture and helps the culture to permeate and purify the learning that is gathered, is the best and most fruitful.

It is the duty of every countryman to assimilate and appreciate the historical and cultural background of his nation.

Education must enthuse youth to understand their precious heritage, culture and spirituality and to evoke the higher powers they possess.

The first step on this journey has to be the inner understanding and awareness that all men are one and the same. There may be differences in colour, race, religion, and physical form, but these are like the different waves on the surface of the mighty ocean.

believe no connection exists between mathematics and culture. Failing to consider other possibilities, they believe that mathematics is "acultural", a discipline without cultural significance." (Barta, 1995, p.12)

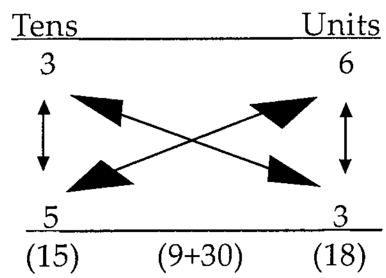
"Children seldom are taught that several of the ancient Greek mathematicians, Pythagoras and Thales (legendary founder of the Greek mathematics) for instance, travelled and studied in places such as India and Northern Africa where they acquired much of their mathematical knowledge. Students know little of the mathematical inventions or applications of such ancient non-European cultures as the Egyptians, the Babylonians, the Mayan, the Incas, to name but a few. They don't understand because they have not been taught that many cultures have contributed to the development of mathematics; cultures whose members were certainly intelligent, resourceful and creative....Mathematics is a compilation of progressive discoveries and inventions from cultures around the world during the course of history. Its history or ethnography has been a wonderful mosaic of cultural contributions." (Barta, 1995, p.13).
Nelson, Joseph and Williams (1993) give examples of selected mathematics topics, showing different approaches to these topics which have been developed in different cultures. One

example they use is multiplication. Children can be shown some of these different methods, to appreciate that there are many different ways of arriving at the same answer. Some samples of the different approaches they describe are summarised below, but it is recommended that you refer to their book for some more:

Mathematics Topic: Multiplication

Vedic Multiplication (India)

36 x 53



Multiply the numbers in the units column vertically (6 x 3 = 18).
Multiply the tens and units columns crosswise and add (i.e. 3 x 3 + 5 x 6 = 39).
Multiply the numbers in the tens column vertically (3 x 5 = 15).
Place value adjustments are made by "carrying over" the relevant numbers, so 15 39 18 becomes

15 39 8 i.e. 15 40 8, then 15 0 8, i.e. 1908.
1 4

from Nelson, Joseph and Williams (1993), p.107

Egyptian Multiplication

28 x 13

Choose one of these numbers (e.g. 28) as the multiplier and continue to multiply it by 2 until you reach a stage where the next number in the left-hand column would exceed the other number (i.e. 13). Find the numbers in the left-hand column which add up to 13 (shown with * below). Then find the total of their corresponding numbers in the right-hand column, to get the answer .

*1	28
2	56
*4	112
*8	224
1+4+8=13	28 + 112 + 224 = 364

from Nelson, Joseph and Williams (1993), p.98

Russian Multiplication

225 x 17

This method involves continually doubling one of the numbers (17) and halving the other (225) but leaving out any remainder. This process continues until the number that is being halved becomes 1. Any row with an even number in the left-hand column is then crossed out, and the remaining numbers in the right-hand column are added together to get the answer.

225	17
~~112~~	~~34~~
~~56~~	~~68~~
~~28~~	~~136~~
~~14~~	~~272~~
7	544
3	1088
1	2176
	3825

from Nelson, Joseph and Williams (1993), p.99

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Teachers who will promote qualities of mutual love and regard in their students are sorely needed today.

Appreciating The Beauty Of Mathematics

The aim of Sathya Sai Education in Human Values is the all-round development of the child Sathya Sai Baba says: "Wisdom flashes like lightning amidst the clouds of the inner sky; one has to foster the flash and preserve the light. That is the true sign of the 'educated' person." It is for this reason that creative work or artwork should become an integral part of the EHV programme. The following are some of the benefits of teaching creative work to children: to activate creativity, to stimulate imagination, as a means of expression, for relaxation and enjoyment, to build self-confidence, to encourage co-operation, to teach discrimination, to encourage responsibililty, to strengthen concentration, to develop co-ordination, to bring out inner talents, to encourage discipline, patience and perseverance and to develop skills.

Naidu (1986) talked about the truth, power and beauty of mathematics. In doing so, he quoted two mathematicians (p.5). The first of these is Plato:

> "In every man there is an eye of the soul which by other pursuits is dimmed and lost; but by mathematics it is purified and reilluminated. This eye of the soul is more precious than ten thousand bodily eyes, for by it alone truth can be seen."

The second quotation is from the seventeenth century astronomer, Johannes Kepler:

> "The chief aim of all investigations of the external world should be to discover the rational order and harmony which have been imposed on it by God, and which revealed to us in the language of mathematics."

Naidu also wrote (p.6):

> "Tilak was a great Indian national leader who was originally a mathematics teacher. Once, an admirer of Tilak approached him and asked: 'Sir! when we gain independence, would you like to become prime minister or president of our nation?' 'Neither', said Tilak, 'I wish to go back to my original position as mathematics teacher. As a teacher of mathematics I can make many disciplined and dedicated ministers and presidents'. To sum up, mathematics is a wonderful subject which has a great relevance in taming the mind, training the intellect, and purifying the vision through the awakening of intuition of the individual. Every mathematics teacher must make it his mission to unfold fully all the intellectual and intuitive facilities of the student by imparting the subject in a proper perspective so as to kindle *love* for the subject, make the student grasp *truth*, create intense *delight* in his heart by unfolding the *power, beauty* and *social value* of the subject of mathematics.

Mathematics Topic: Number Patterns, Simple Algebra

Fibonacci Numbers

The Fibonacci numbers, discovered in the thirteenth century by Leonardo Fibonacci, show how mathematics connects seemingly unrelated things (Barnard, 1996).

One way to begin a study of the Fibonacci sequence of numbers is to start with a pair of rabbits (one male, one female). Rabbits begin to produce young two months after their own birth. After the first two months, each pair produces a mixed pair (one male, one female) and continues to produce another mixed pair each month. Students can count the number of pairs born in each month, finding that the sequence will be 1,1,2,3,5,8,.... Soon they can find a way to predict subsequent terms in the series.

Barnard (1996) listed several examples of Fibonacci numbers in Nature: "Cutting a bell pepper crosswise reveals 3 chambers. An apple has a 5-point-star cross section, and a lemon has an 8-chambered cross section. A daisy almost always has 13, 21 or 34 petals. Sunflower seeds spiral out from the center with 21 spirals in one direction and 34 in another. The giant sunflower has 89 amd 144 spirals, and the shopper sunflower has 144 and 233 spirals. Each set of spirals contains adjacent Fibonacci numbers." (p.1).

Students can be asked to find the ratios of successive Fibonacci numbers. This will give the golden ratio, which was the basis for the golden rectangle used in ancient Greek architecture to achieve perfect proportions in structures such as the Parthenon (Pappas, 1987). The golden rectangle is also evident in the proportions of the human body, as illustrated by Leonardo da Vinci (Pappas, 1987).

Garland (1987) pointed out that Fibonacci numbers and proportions appear in musical scales, and in the division of musical time in compositions. Fibonacci proportions have been found in compositions including Gregorian chants, Bach fugues, and Bartok sonatas. "It has been suggested that the Fibonacci numbers are part of a natural harmony that not only looks good to the eye but sounds good to the ear" (p.34).

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SOME FURTHER EXAMPLES OF INTEGRATING EDUCATION IN HUMAN VALUES INTO MATHEMATICS TOPICS

Other suitable themes that have been used to integrate education in human values into mathematical problems, and some extensions of these themes are:

- Proportions of areas planted with different kinds of seeds: As an extension activity, plant some fast-growing seeds (eg beans). Put some in the classroom where you have your music and silent sitting (PEACE). Put some, as a control, in another classroom (with the same amount of light and water) where there is no music or silent sitting. You can also ask the children during their silent sitting to concentrate on sending loving feelings to one of the plants but not to the others (LOVE). Compare the growth of the different plants over time. What do the results tell us about <u>people's</u> needs?

- Budgeting percentages of pocket money for different purposes (RIGHT ACTION): As a supplementary service project, children can be introduced to the value of "Ceiling on Desires" – that is to develop self-discipline and self-sacrifice rather than expecting to get everything we want. Each child can be asked to sacrifice something that they think they want (e.g. buy less sweets for a month) and discover how much money they save. The teacher can also participate. At the end of the month, the money they have saved can be used to help somebody in need. Discuss with the children how this makes them feel.

- Percentage: One teacher introduced the percentage lesson with the scenario, 'In a class of 100 pupils, 17 thought of themselves as truthful children (ie always told the truth and never told a lie). In a class of 200 children, 20 said they were truthful.' As an extension, he did a survey with the children in his class, in which he gave them a list of values and asked them to think carefully and honestly about which of these values applied to themselves. The survey data were used to develop the percentage activities and at the same time encouraged them to self-reflect about their own good qualities or where they need to improve.

Some more examples of teachers' ideas for incorporating values education into existing mathematics topics are shown below.

Topic:
Measurement without the use of units: direct comparison, e.g. comparing children and objects according to height (taller, tallest; bigger, biggest)
Values for discussion with pupils:
Does it always mean that the biggest is 'better' than the smallest? What is the truth about this? Is the one who is the tallest in the class necessarily the one who is the heaviest? has the longest armspan? the one who can run the fastest? the one who can hold his/her breath the longest? ie we all have some special thing at which we are outstanding. We need to find that special thing and use it wisely. We don't need to be jealous of others because everyone's outstanding thing is different. (NON-VIOLENCE/respect for diversity)

Topic:
Measurement: Comparison with informal units
Values for discussion with pupils:
When we measure with informal units it is difficult to make comparisons because the units are

all different. The same applies to our daily lives. If we all do our 'own thing' it is difficult to compare and we have chaos–how does this relate to our lives in general? (LOVE/interdependence)

Topic:
Measurement: Comparison with formal units
Values for discussion with pupils:
We need standards/rules so we can all have a unified concept, common ways of looking at things. What are some real-life examples that support this idea? (LOVE/ interdependence)

Topic:
Simple and compound interest, percentage
Values for discussion with pupils:
Some financial advisors suggest that we should immediately save 10% of any income that comes to us before we spend any of it. If you have a monthly income of [insert amount] and you save 10%, and invest it at an interest rate of [insert rate], how much will you have after 1 year? After 10 years? They also suggest that 10% is a small amount that we will not miss, so we can afford to give away 10% of everything we earn to those who are needy. Over a period of 10 years how many people do you think you could help by giving away 10% of all your earnings? (RIGHT ACTION/efficiency, NON-VIOLENCE/benevolence)

Topic:
Division by zero
Values for discussion with pupils

$$\text{Happiness} = \frac{\text{Number of desires fulfilled}}{\text{Number of desires entertained}}$$

Sathya Sai Baba

Ask students to discuss what happens when the number of desires entertained is zero–i.e. that is when we achieve infinite happiness (PEACE/contentment, self-discipline)

Topic:
Games to practise number facts.
Example: Sit in a circle and hold hands. As each child has his/her turn the previous child passes on the pulse of loving energy by squeezing his/her hand and sending support that s/he will get the right answer. Start with a number fact, eg 2x4. The next child answers 8, then continues with a new fact, eg (... x 2 = 16 – note that they do not repeat 8 as part of the game is to listen carefully to the number the first time it is said). The game continues until everyone has had a turn. Conclude by having the children talk about their experiences with the game.
Values for discussion with pupils
If we send love and support to other people to help them to get the right answer, rather than being jealous or hoping they make a mistake, it makes them feel good and it also makes us feel good (LOVE/compassion, selflessness).

Games that involve an element of skill and an element of chance promote discussion about:
TRUTH/honesty – not cheating by changing numbers or pretending you have an answer if you don't,

PEACE/self-esteem – everyone has an equal chance in life as long as they put in the work (i.e. to know the answers to the questions).

If some children are clearly disappointed that they do not get a chance to win, this provides a chance to talk about how they feel if somebody else wins and not themselves (jealousy) – how jealousy can damage us, not the other person, so it is better if we feel happy for the person who is successful.

Topic:
Equivalence of fractions
Values for discussion with pupils:
We can have different names and appearances, but we are all the same (NON-VIOLENCE/ respect for diversity).
After pupils have looked at the patterns in equivalent fractions, tell the story of Fraction Land and have them make up their own stories using equivalent fractions, that bring out the underlying value.

> Once upon a time in Fraction Land there was chaos. All the fractions were constantly arguing with each other. 6/12 thought he was the best because he had the biggest numbers on both the top and the bottom, so he was cruel to the fractions with smaller numbers, like 2/4 and 3/6. 4/8 would not allow his children to play with the children of 3/6 because they always cut their food into 6 pieces and 4/8 believed that the only proper way was to cut it into 8 pieces – he did not want his children to learn any "wrong" ways of doing things. Mrs. 2/4 would shout at Mrs. 5/10 in the market because Mrs. 5/10 always bought 5/10 of a kilogram of everything and Mrs. 2/4, who only ever bought 2/4 of a kilogram, thought she was being very greedy. The 2/4 family would always cross the street if they saw anyone from the 3/6 or 4/8 families approaching them because they looked so different that they thought they must be very bad people indeed.
>
> Things became so bad that the fractions all began to pray to their own gods to make their enemies go away. It was no longer a safe or happy place to be. One day a brilliant bright light appeared, and out of it stepped the most beautiful angel the fractions had ever seen. Her dress was painted with golden numbers that said 1/2. Suddenly everything went dark and when the lights came on again the fractions looked at each other in great surprise. They all looked exactly the same as each other. Everyone's tops had turned into a 1 and the bottom parts had turned into a 2. What had happened? Were they still the same Fraction Land people they had always been? Or had the angel turned them into different people?

Topic:
Addition and subtraction of fractions
Values for discussion with pupils
What have we learned from this rule that tells us fractions and people have the same needs? Fractions need to be the same family before we can add or subtract. It is only by becoming "one family" that we can work together. Ask children to suggest some real-life examples to illustrate this. (LOVE/interdependence, NON-VIOLENCE/respect for diversity).

Topic:
Logarithms
Values for discussion with pupils:
Illustrate the method of logarithms by considering the sequence of powers of 2.
If one person (2^0)has love and peace in his/her heart and influences just one other, that will be 2 (2^1). If they each influence one other it will be 4 (2^2), then 8 (2^3), then 16 (2^4) etc. Graph these and show the effect of the exponential increase. We all think that we, as individuals, cannot make an impact, but this illustrates that if we change ourselves, "one candle can light many lamps" (PEACE, LOVE)

Topic:
Basic algorithms
Values for discussion with pupils:
Commutativity, eg

> 2x3 = 6, 3x2 = 6
> They may look different but they are the same. Pupils can be encouraged to make lists of things that are the same about all people despite the fact that they look different.

Number sentences, eg

> 5+3+2+1=11
> 2+3+5+1=11
> 3+1+5+2=11
> 4+6+1=11
> 2+7+2=11
> 3+3+5=11
>
> There are many different ways of arriving at the same answer. What does this mean in relation to people? People may do things in different ways but we should not accuse them of being wrong if their way is different from ours. This discussion can be extended to cultural and religious differences between people (NON-VIOLENCE/respect for diversity)

Topic:
Equations
Values for discussion with pupils:
This teacher began with a silent sitting exercise in which the pupils were asked to visualize a village which was at first surrounded by trees but which changed as the trees were cut down and was eventually destroyed by a flood because there were no trees to protect the soil. The children were asked to think for a moment about, "What can I do right now to prevent this problem?" and what they had learned from reflecting on this scene. This helped them to realize that everyone has a responsibility to the environment, not just to leave it to others.

The theme of balancing the environment was used to lead into the topic of balancing equations. It could also be related to balance in themselves, i.e. keeping themselves peaceful even when things go wrong, and how they can get back into balance if they feel disturbed or bothered by something (e.g. breathing deeply, doing silent sitting, having a drink of water and lying down for a while). (PEACE/calmness, equanimity, NON-VIOLENCE/concern for ecological balance).

Topic:
Area of a parallelogram: cutting and rearranging the parallelogram to form a rectangle
Values for discussion with pupils
This is an example of Truth (i.e. "that which never changes), that the area will remain the same no matter how the shape is rearranged. It is good for children to develop a sense that some such things have held true forever and will continue to be true forever. Can they think of any other examples of this kind of Truth? (TRUTH)

Topic:
Calculation of income tax
Values for discussion with pupils
From where does the government get funds? The government provides roads etc. For us, so we have some responsibility/moral obligation to give something in return, i.e. take taxation as a moral duty towards the society that is doing so much for us, not as a burden.
Talk about tax evaders and the effects of increasing the taxes/increasing the burden of the salaried people, therefore the importance of co-operation (RIGHT ACTION/dependability, NON-VIOLENCE/respect for property).
Regarding concessions for money given in donations: We are not getting anything out of it so why should we donate? We should give from the heart because we want to and because it is going to benefit some people who we may or may not know. We are not here for ourselves only, we have duties to others. If you don't have money you can do it with your hands, but when it comes to income tax the government encourages you to give money for a good cause. This is the concept of selfless service. Don't be stingy with the amount of money you are prepared to part with, compared to being prepared to spend a lot of money for our own purposes (LOVE/compassion)

Topic:
Properties of a circle
Values for discussion with pupils:
What is the difference between the circle and other shapes? A special property of the circle is that it is complete. We need to try to be like the circle – strong and complete in ourselves and not needing to rely on outer things to make us happy.

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Education today needs to impart to the students the capacity or grit to face the challenges of daily life.

SECTION 3

Teaching Human Values Through Examples Of Great Mathematicians

Are we teaching to our students the exemplary lives of those who struggled for freedom and gave even their lives for the sake of the country? Are we teaching to our students the message of great men who proclaimed the supremacy of morality and character?...no greater virtue than refraining from harming others, compassion, courage, sacrifices.

Famous historical figures whose actions have reflected human values should be presented as edifying examples for children. Stories about historical persons should be related even when they have negative connotations, for lessons can be learnt from all types of situations. Children should not just memorize facts and dates, but should discover the values from everything in history.

Many of the greatest scientists, musicians, poets and artists have attained their inspiration from the depth of silence.

Students now believe that the study of books is all that is needed. But, what is the result to be gained? The test for scholarship is: are soft and sweet words uttered? are good works planned and executed? are scholars involving themselves in doing good for the society? are they avoiding acts that injure others? are they grateful to those who promote their happiness?

Have high aims in life. Set before yourselves the examples of great men and women who have figured in the history of your country and the world. Take a lesson from their life of sacrifice and heroism.

Let them know and be told that history is replete with stories of achievement of men and women who fought fearlessly against overwhelming and frightful odds of poverty and/or illness, to acquire knowledge and skill. Children must be guided into the channel of emulating such persons.

"Using biographies of mathematicians can successfully bring the human story into the mahematics class. What struggles have these people undergone to be able to study mathematics?..." (Voolich, 1993, p.16)

SIR ISAAC NEWTON

"We all have something within us which helps us, guides us, gives us the conscience to know what is right and wrong. This "something" also gives us knowledge and wisdom. Whenever we cannot think of a solution to a problem we sit still and calm our mind. Very often the answer will come in a moment of intuition. Sir Isaac Newton, after thinking for some time on the effect of gravity, could not solve the problem. So Newton went for a walk to relax and when sitting quietly under an apple tree, saw an apple fall down; in a flash of understanding Newton understood the law of gravity which governs the movement of minute particles as well as the stars and planets. Many great scientific discoveries have been made not during serious thinking or when doing a lot of calculations but while the mind is relaxed. This is when intuition starts."

This quotation was taken from
Education in Human Values Handbook for Teachers, p.11.

PYTHAGORAS - Born about 580 B.C. in Samos, Greece.
Examples of Contribution to Mathematics:
odd and even numbers, harmonic progression in the musical scale, geometry

> The ancient society of Pythogoreans practised humility. One of their beliefs was that, to show their humility, they should never take a higher road if a lower one was present. (Pappas, 1987) Hooper suggested that one of the reasons why Pythagoras and his followers progressed was that they practised community living and sharing of resources. As well as sharing their worldly goods, they also shared their mathematical and philosophical discoveries.
>
> *(Hooper,1948)*

MARIA AGNESI - (1718-1799) Italy
Example of Contribution to Mathematics: calculus

> "Maria was a child prodigy, but was also shy. She stayed at home, teaching the younger children and following her own studies. When her mother died after giving birth to twenty-one children, Maria took over the running of the household. At the age of twenty she started a ten year project, a book bringing together the work on calculus of Leibnitz and Newton titled Analytic Institutions. Sometimes she would have trouble with a problem. But her mind went on working even in her sleep; she would sleep-walk to her study and back to bed. In the morning, she would find the answer to the problem waiting on her desk. Her book made her famous; she was living proof of what she had argued at nine years old [that women had a right to study science]. But Maria had other interests in her life apart from mathematics. She had always worked with the poor people in her area, and she had asked her father for separate rooms and turned them into a private hospital. She worked at the hospital (and another) until she died at the age of eighty-one. Maria Agnesi wrote an important book on mathematics, as well as another unpublished book. She ran a household of over twenty people, and she worked for people who had not had her luck and opportunities. Each one of these things was remarkable, but she did them all."
>
> *(Lovitt and Clarke, 1992, p.560)*

RENE DESCARTES - Born, March 31st, 1596 at La Haye, near Tours, France
Examples of Contribution to Mathematics: analytic geometry; application of co-ordinate geometry to equations of curves

> Descartes lived during a troubled time, when strong rulers and politicians took whatever they wanted from whomever they wanted, by force. It was also a time of "religious bigotry and intolerance which incubated further wars and made the dispassionate pursuit of science a highly hazardous enterprise" (p.35). Added to this was a prevalence of plague and disease brought about by the lack of cleanliness and proper sanitation just as common amongst rich people as poor. Descartes overcame these hardships to succeed in his chosen field. Because Descartes was an unhealthy child, his teacher allowed him to lie in bed for as long as he wanted to in the mornings. From this, Descartes learned of the value of quiet meditation whenever he needed to think, and continued to do this for the rest of his life. This was the time when his great mathematical and philosophical discoveries came to him. One famous quotation of Descartes' was that he preferred truth to beauty. Descartes was not wealthy, but he was contented with what he had and felt that this was enough. He was moderate in his habits, and was very kind to others. While he often inflicted a Spartan way of living on himself, he never expected the rest of his household to live in the same way. He continued to help with the welfare of his servants long after they had left his service.
>
> *(Bell, 1937)*

LEONARD EULER - Born, April 15th, 1707 at Basle, Switzerland
Examples of Contribution to Mathematics: calculus of variations, analysis of rotations, fundamental equations of fluid motion used in hydrodynamics, convergence of series

> The first extract is not about Euler himself, but about his servant, Peter Grimm. While living in St. Petersburg, Euler was a victim of the great fire of 1771, when his house and all of its contents were destroyed. Grimm risked his own life to save not only Euler, but some of his valuable manuscripts. Towards the end of his career, Euler developed cataracts and eventually became totally blind. Rather than becoming bitter about his affliction, he accepted it with good faith, and made the most of his situation and his mathematical productivity increased rather than diminished. While he still had some sight, he would write his formulas with chalk on a large slate, and then dictate the words explaining the formulas.
>
> *(Bell, 1937)*

PIERRE-SIMON DE LAPLACE - Born, March 23rd, 1749 in Beaumont-en-Auge, France
Examples of Contribution to Mathematics: astronomy (e.g. the mean distances of the planets from the Sun are invariable to within certain slight periodic variations); theory of probability; theory of the potential (which had the effect of replacing partial differential equations in two or three unknowns by equations in one unknown).

> It has been recorded that Laplace had many faults, "greed for titles, political suppleness, and desire to shine in the constantly changing spotlight of public esteem" (p.172), and he often stole from the works of his contemporaries and predecessors. Yet in spite of these, "Laplace had elements of true greatness in his character" (p.172). He was very generous to unknown beginners in the field of mathematics and, to help one young man, actually cheated himself. He also showed great moral courage when his true convictions were questioned, and even had the nerve to stand up to Napoleon and tell him the truth, when others were being reduced to tears by Napolean's deliberate brutality.
>
> *(Bell, 1937)*

JOHANN FRIEDERICH CARL GAUSS - Born, April 30th, 1777, in Brunswick, Germany
Examples of Contribution to Mathematics: non-Euclidean geometry; congruence [e.g. if the difference (a - b or b - a) of two numbers a, b is exactly divisible by the number m, we say that a,b are congruent with respect to the modulus m]; method of least squares; theory of binary quadratic forms; every positive integer is the sum of three triangular numbers:

> Gauss lived in turbulent times, and was horrified by the Paris revolt of1848. "He saw peace and simple contentment as the only good things in any country" (p.258). "Another source of Gauss' strength was his scientific serenity and his freedom from personal ambition. All his ambition was for the advancement of mathematics" (p.259). Gauss showed great caring for his mother. " The last twenty two years of her life were spent in her son's house and for the last four she was totally blind. Gauss himself cared little if anything for fame; his triumphs were his mother's life. There was always the completest understanding between them, and Gauss re-paid her courageous protection of his early years by giving her a serene old age. When she went blind he would allow no one but himself to wait on her, and he nursed her in her long last illness." (p.220). Gauss was modest, and reticent to boast about his work. "All his life, there was but one type of man for whom Gauss felt aversion and contempt, the pretender to knowledge who will not admit his mistakes when he knows he is wrong" (p.231).
>
> *(Bell, 1937)*

SONJA (SOPHIE) KOWALEWSKI - Born, January 15, 1850, in Moscow, Russia
Examples of Contribution to Mathematics: won the Bordin Prize of the French Academy of Sciences for her memoir *On the Rotation of a Solid Body About a Fixed Point*

Madame Kowalewski was one of the leading women mathematicians of modern times, and was also famous as a leader in the movement for the emancipation of women. For five years, she studied diligently and became well-respected and famous. Then she allowed herself to become "the focus for the flattery and unintelligent, sideshow wonder of a superficially brilliant mob of artists, journalists, and dilettante literateurs who gabbled incessantly about her unsurpassable genius. The shallow praise warmed and excited her" (p.427). She was so busy with this new life that she stopped studying mathematics. Fortunately, after the birth of her daughter, she returned to her job, and went on to make great contributions to the field of mathematics.

(Bell, 1937)

GEORGE BOOLE - Born, November 2, 1815, in Lincoln, England
Examples of Contribution to Mathematics: discovered invariants, created a simple, workable system of symbolic or mathematical knowledge

As a young child, Boole realised that he needed to know Latin and Greek if he was to succeed in his chosen field. Because his family was poor, he was unable to attend the schools where he could learn these, so he decided to teach himself. He showed great persistence, and had mastered both by the age of 12. At the age of about seventeen, as Boole was walking across a field, it "flashed upon him...that besides the knowledge gained from direct observation, man derives knowledge from some source undefinable and invisible" (p.447).

(Bell, 1937)

CHARLES HERMITE - Born, December 4, 1822, in Lorraine, France
Examples of Contribution to Mathematics: Abelian functions, trigonometry (functions of two variables and four periods), algebraic invariants, general equation of the fifth degree, transcendental numbers

"Great as were Hermite's contributions to the technical side of mathematics, his steadfast adherence to the ideal that science is beyond nations and above the power of creeds to dominate or to stultify was perhaps an even more significant gift to civilization .... We can only look back on his serene beauty of spirit with a poignant regret that its like is nowhere to be found in the world of science today. Even when the arrogant Prussians were humiliating Paris in the Franco-Prussian war, Hermite, patriot though he was, kept his head, and he saw clearly that the mathematics of 'the enemy' was mathematics and nothing else....To Hermite it was so obvious that knowledge and wisdom are not the prerogatives of any sect, any creed, or any nation ..." (p.465).

(Bell, 1937)

ERNST KUMMER - Born, January 29, 1810, in Brandenburg, Germany
Examples of Contribution to Mathematics: discovered "ideal" numbers, continued Gauss' work with quadratics, complex numbers composed of roots of unity

"Remembering his own struggles to get an education and his mother's sacrifices, Kummer was not only a father to his students but something of a brother to their parents. Thousands of grateful young men who had been helped on their way by Kummer at the University of Berlin or the War College remembered him all their lives as a great teacher and a great friend. Once a needy young mathematician about to come up for his doctor's examination was stricken with smallpox and had to return to his home in Posen near the Russian border. No word came from him, but it was known that he was desperately poor. When Kummer heard that the young man was probably unable to afford proper care, he sought out a friend of the student, gave him the requisite money and sent him off to Posen to see that what was necessary was done." (p.515).

(Bell, 1937)

HENRI POINCARE - Born, 1854, in France

Poincare was interested in the psychology of mathematics, and particularly in how mathematicians make their discoveries. He believed in the importance of the subconscious mind, and wrote the following illustration (pp.550-551). "For fifteen days I struggled to prove that no functions analogous to those I have since called Fuchsian functions could exist.... Every day I sat down at my work table where I spent an hour or two; I tried a great number of combinations and arrived at no result. One evening, contrary to my custom, I took black coffee; I could not go to sleep; ideas swarmed up in clouds; I sensed them clashing until, to put it so, a pair would hook together to form a stable combination. By morning I had established the existence of a class of Fuchsian functions .... I had only to write up the results, which took me a few hours....I then undertook the study of certain arithmetical questions without much apparent success....Disgusted at my lack of success, I went to spend a few days at the seaside and thought of something else. One day, while walking along the cliffs, the idea came to me." Because of these experiences, Poincare became very interested in the theory of the subconscious mind and its part in creating mathematics. "Honours were showered upon him by all the leading learned societies of the world, and in 1906, at the age of fifty two, he achieved the highest distinction possible to a French scientist, the Presidency of the Academy of Sciences. None of all this inflated his ego, for Poincare was truly humble and unaffectedly simple. He knew of course that he was without a close rival in the days of his maturity, but he could also say without a trace of affectation that he knew nothing compared to what is to be known." (p.553)

(Bell, 1937)

QIN JIUSHAI - Born ca.1200 AD in Sichuan, China
Examples of Contribution to Mathematics: solution of simultaneous linear congruences, measuring the area of fields, surveying (right-angled triangles and double difference problems), taxation problems, transportation of foodstuffs and capacities of warehouses, problems on architectural constructions

Qin wrote "I believe all things can be accounted for by Numbers and so I have been involved in searching for truth, seeking discussions with the learned ones, enquiring into the unknown and now I have a rough idea...[I am] attempting to formulate it in terms of questions and answers so as to make it available." (p.112).

Li Yan & Du Shiran (1987)

ZU CHONGZHI - Lived 429-500 AD in northern China
Examples of Contribution to Mathematics: astronomy, calendar making, inventions of machinery e.g. a boat fast enough to cover a great distance in one day, theory of music, calculation of Pi to 7 decimal places

> "When Zu Chongzhi was 33 years old, he completed the construction of a new calendar, called the Da Ming. Judging from the scientific knowledge of that period it was the best calendar available. However, the new calendar provoked a lot of objections from very important influential people in the court such as Dai Faxing. Because of fear of the influence of Dai Faxing most of the court officials were afraid to give an unbiased opinion of the new calendar of Zu Chongzhi. In order to defend the truth, Zu Chongzhi started a public debate with Dai Faxing....[As part of this debate he] wrote two very famous lines... 'Willing to hear and look at proofs in order to examine truth and facts' and 'floating words and unsubstantiated abuse cannot scare me'. He hoped that both sides would present concrete evidence to determine the rights and wrongs and he was not at all afraid of unfounded rumours and abuse." (p.81)
>
> Li Yan & Du Shiran (1987)

MARY SOMERVILLE - Born 1780 in Burntisland, Scotland
Examples of Contribution to Mathematics: algebra, differential and integral calculus

> Mary was one of the world's first famous female mathematicians. She became interested in mathematics, and desperately wanted to study it, at a time when it was not considered acceptable for a woman to do so. She bought books on algebra and geometry and read them at night. Despite disapproval from the people around her, she persisted with her struggle to learn. Later in her life she began to solve problems in a magazine, and won a prize for her solution to an algebra problem. She went on to write several books about mathematics and science. Later in her life, she reflected on "the long course of years in which I had persevered almost without hope. It taught me never to despair" (p.6). "Mary Somerville used an approach to her work that is useful today. If she couldn't find the key to unlock a difficult problem she stopped work and turned to the piano, her needlework, or a walk outdoors. Afterward, she returned to the problem with her mind refreshed and could find the solution. If she could not understand a passage in her reading, she would read on for several pages. Then, going back, she could often understand what was meant in the part which had been confusing" (p.12).
>
> *Perl (1993)*

MARY EVERETT BOOLE - Born 1832 in England and lived in Poissy, France as a child
Examples of Contribution to Mathematics: geometry of angles and space; string geometry (curve stitching), mathematical psychology (understanding how people learn mathematics)

> As a young girl, Mary was very compassionate towards animals. Perl reported that she frequently rescued insects that had been hurt by frost or rain, and nursed them back to health. As an adult, she worked as a librarian in a women's college, and showed the same compassion in becoming a friend and mentor to the students. She invited students to discussion sessions about mathematics and science, and one of these students later wrote; "I found you have given us a power. We can think for ourselves, and find out what we want to know" (p.50). Even as an old lady, during World War I, Mary opened her house to people who needed to "find a quiet place for an hour, away from the turmoil of a country at war and the terrible news in the newspapers" (p.55).
>
> *Perl (1993)*

FRANCOIS VIETTA - Born 1540 in Poitou, France.
Example of Contribution to Mathematics: symbolism in algebra

> "Vieta was a kindly and generous person, as is shown by the fact that he once entertained a scientist for several weeks, although that scientist opposed his opinions. He even paid his visitor's travelling expenses. His generous unselfishness is shown in the way he would send copies of the mathematical papers he wrote to scholars all over Europe" (p.101).
>
> *Hooper (1948)*

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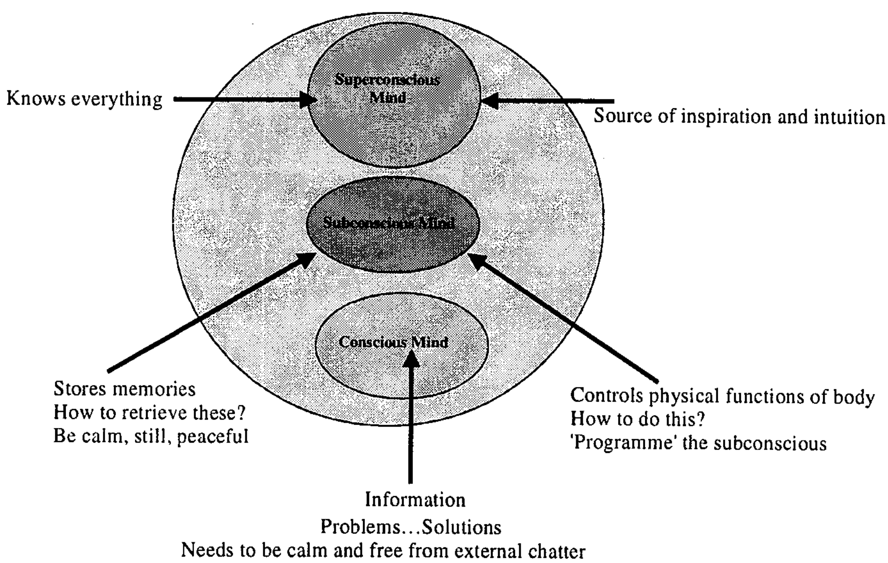
"If the educational system could contribute to the turning out of good character, committed to human values, the country will become stronger and greater as a nation and be a model to the world."

Section 4

Mathematics And "Silent Sitting"

We all know that teaching, these days, is becoming increasingly difficult due to increasing problems with discipline, lack of concentration by pupils, and lack of pupil motivation. Just as life is becoming more difficult for teachers, it is also becoming more difficult and complicated for pupils. Therefore, for the sake of both their own well-being and that of their pupils, teachers are constantly searching for ways to address these problems. The purpose of this section is to suggest some ways in which the techniques of "silent sitting" and "creative visualisation" can be utilised in the mathematics classroom without taking too much time away from other activities. Neither of these techniques needs to take up a lot of time in the classroom - just a few minutes once or twice a day are enough.

The value of silent sitting and creative visualisation was illustrated in Jumsai's (1997) model shown below.



This model considers the three levels of the mind: the conscious, the subconscious and the superconscious. Through the five senses, the conscious mind receives and processes information from the environment in order to create awareness and understanding. The subconscious stores the memories of everything that we have experienced, and feeds these memories to the conscious mind to control the individual's thoughts and actions, and even to color our perceptions of events that happen around us. The superconscious mind is the source of our wisdom, knowledge, conscience and higher consciousness. In a holistically-balanced person, these three levels of the mind interact together to contribute to the physical, mental, emotional and spiritual well-being. Jumsai proposes that there are two important ingredients for this healthy interaction to occur. The first is to free the three levels of the mind from extraneous 'chatter', to enable enhanced concentration and memory. The second is to ensure that the information that is stored in the various levels of the mind is 'clean', positive and constructive, since its retrieval will have such a significant effect on the individual's thoughts and actions which in turn contribute to the presence or absence of holistic well-being. The technique of silent sitting is a useful way to quieten the chatter and hence promote a feeling of inner peace, and that of creative visualisation can programme the mind in a positive, healthy way.

Children need to have time to just sit and get into contact with their inner selves if they are to be able to improve their concentration and maintain balanced physical, mental and emotional health. Children only need to practise silent sitting and inner listening for a few minutes each day to be able to experience its benefits. Five or ten minutes is usually quite enough. Most children appreciate the opportunity to listen to their own inner silence for a while. If they do not find this easy to do, you can help them by playing some soft music that will give them a focal point. In time they will experience the sense of inner calmness and the music will not be as important.

REPORTED BENEFITS OF USING SILENT SITTING AT THE BEGINNING OF A LESSON

Several studies have explored the effects of utilising techniques such as those described above regularly in the classroom. In particular, positive benefits have been derived for disruptive or inattentive pupils (Bealing, 1997). One study found that it helped to improve their decision making and put them in touch with their deeper core values (Rozman, 1994), while another found that it helped them to cope better with stressful events (Rickard, 1994). Further benefits have included decrease in levels of impulsivity, increase in attention span and general improvement in behaviour (Kratter and Hogan, 1982). In the UK, Anita Devi found with her Grade 4 class that regular use of silent sitting led to a significant improvement in the children's concentration, behaviour and mathematics attainment. When, as a control, she stopped the practice for a month there was a decline in all three of these aspects, and eventually the children themselves asked to start it again.

In other research projects around the world where mathematics teachers have utilized silent sitting at the beginning of lessons, children have typically made comments such as the following:

- √ It can help me to recall previous knowledge and help me to learn new things.
- √ It is easier to concentrate and remember what I have learned.
- √ I am not very easily annoyed or irritated any longer.
- √ It helps me to forget sad things.
- √ I can actually find a way to solve a problem by using silent sitting.
- √ It is quicker to think of answers than with eyes open. Time slows down when my eyes are closed, so there is more time to find a solution.
- √ I have found myself to be more energetic and clear-minded.
- √ I have found more interest in learning and study than before. Now I enjoy studying.
- √ I have made obvious progress in my study since using silent sitting.

In the classroom, it is possible to help pupils to develop strategies for programming their subconscious minds in positive, constructive ways. The following are examples of some visualisations that teachers have led their children through in a relatively short time during silent sitting.

VISUALISATIONS FOR STARTING A MATHEMATICS ACTIVITY

Visualisation 1
Close your eyes and take some slow, steady breaths. Think very hard about the part of your brain where your mathematics skills are kept. Think of that place in your brain as being like a flower. As you breathe in, imagine that the breath is caressing the flower like a soft gentle breeze. As it touches, the flower starts to open slowly, petal by petal, until it is fully open. This flower is your potential to understand mathematics and to do the problems. Now that the flower is open you will find that the mathematical thinking will come to you quickly and easily. Open your eyes now and you can begin your work.

Visualisation 2
Close your eyes and imagine that there is a candle burning inside your head. Let the light get brighter and brighter until it fills your whole head. Let it light up your brain so that you will be able to think clearly and well. Imagine that the same light is going from you to everyone in your class, so they will be able to think clearly too.

After this visualization the following points can be discussed with pupils:

- √ You have the knowledge and ability inside your head already.
- √ Regular use of this kind of visualization will help to improve your concentration.
- √ Wishing for classmates what you wish for yourself (i.e. to do well) is more healthy than feeling envy or jealousy.

VISUALISATIONS FOR PROBLEM SOLVING

Visualisation 1
First read the problem. Then put it aside. Close your eyes and just listen to the inner silence of your mind for a few moments. Focus your concentration on the back of your closed eyelids at the point where your eyebrows meet. Don't try to think about anything – just allow your mind to be still and empty, and concentrate on the blankness behind your eyes. When you feel that your mind is completely still, think for a moment about the problem you need to solve. You can either repeat the whole question in your mind, or you can simply say, "I need to find the solution to the problem I am about to tackle." Once you have asked this question, return your attention to focusing on the silent, blank emptiness of your mind behind your closed eyelids for a few more minutes. Then visualise your subconscious mind working like a computer. First it sorts the knowledge you already have to solve the problem. Then it sorts out what else you need to know. Next it puts this knowledge together in a logical way. Finally it sends the output into your conscious mind so it can work on the problem. Take 3 slow, deep breaths, then open your eyes and start to work on the problem.

Visualisation 2
Take 3 deep, slow breaths. Each time you breathe out, let go of any frustration or anxiety. Each time you breathe in, breathe in inspiration. You can decide what this might look like – might be a light that lights up your mind like a bulb, might be a colour, or might be a shape. Just keep drawing it in each time you breathe. Now imagine that your mind has gone completely blank

– as if there has been a power cut and it has been plunged into darkness. Sit there for a few moments in the total blackness. If any thoughts or images come into your head, just let them go and return to thinking about the darkness.
Now imagine that you are going down a long, dark tunnel, right into the deepest part of your mind. This tunnel leads you to your inner mathematician, deep inside your brain. This is the place where you have all the answers and all the techniques you need to solve the problem. All you need to do is unlock the door behind which the inner mathematician is sitting. The door is golden, and in the lock is a big golden key. Slowly turn the key, open the door, and all the knowledge you need can be seen right there. As you return along the tunnel, imagine that you are dragging the knowledge along behind you, bringing it closer and closer to the front of your conscious mind, where you can put it to good use. Now open your eyes – don't worry if the inspiration isn't there immediately, as it will come.

"THE LIGHT VISUALISATION"

In SSEHV a particularly powerful and beneficial form of silent sitting is used with children of all ages as well as adults. The Light Visualisation is in fact fundamental to the SSEHV Programme. It allows the child to progress safely through the three stages described by Sathya Sai Baba as necessary for contacting the superconscious mind: concentration, contemplation and meditation (where meditation simply means the state of being in touch with one's own superconscious mind). The following extract appears in many SSEHV materials but, in this instance, has been taken directly from *The Five Human Values and Human Excellence* by Art-ong Jumsai Na Ayudhya (Bangkok: International Institute of Sathya Sai Education), pp. 83-88.

This is a valuable exercise to do with children on a regular, preferably, daily basis. The light is very important because it is associated with knowledge, wisdom, power and warmth - it literally dispels darkness.

> Imagine that there is a light in front of us. If this is difficult to imagine we may light a lamp or a candle and place it in front of us then open our eyes and look at the flame for a short while. Then we should close our eyes and try to visualise this light. Now using our imagination, bring this light to the forehead and into the head. Let the head be filled with light. Then think, "Whenever there is light, darkness cannot be present. I will think only good thoughts". Now bring the light to the area near the heart and imagine that there is a flowerbud there. When the light reaches the bud imagine that it blossoms into a beautiful flower, fresh and pure: "My heart is also pure and full of love". Now let the light travel down the two arms to the hands. Let these hands be filled with light: "Let me do only good things and serve all". Now the light is moved through the body and down the legs to the feet: "Let me walk straight to my destination, let me walk only to good places and to meet with good people". Now bring the light up to the head once again and leave it there for a little while. Now continue to move the light to the eyes and let our two eyes be filled with light. Again concentrating on the light, think "Let me see the good in all things". Slowly move the light to the ears. Let the ears be filled with light and think, "Let me only hear good things". From the ears we move the light to the

mouth and tongue. "Let me speak only the Truth, and only what is useful and necessary". Now imagine that the light is radiating from your being to surround your mother and father. They are now full of light. "May my mother and father be filled with peace." Now radiate the light to your teachers and send it out to your relatives and friends and especially to any people who you think are being unkind to you. Let it expand out into the whole world to all beings, animals and plants everywhere. "Let the world be filled with light; let the world be filled with love; let the world be filled with peace". Remain immersed in this light and send it out to every corner of the universe and think to yourself, "I am in the light...the light is in me...I AM THE LIGHT"..... Then take the light back to your heart and let it remain there for the rest of the day.

A QUESTION FOR DISCUSSION OR ACTION RESEARCH IN YOUR CLASSROOM

- Use silent sitting at the beginning of your mathematics lesson at least three times per week but preferably every day. Talk to the class about the reasons for doing this.
- Randomly select 6 children. Interview them once a month to monitor their feelings about silent sitting. How do they feel about doing it? Has it had any effects on their behaviour, feelings or academic performance?
- Keep records of changes in your pupils' academic results.
- Keep anecdotal records of any critical incidents, particularly in relation to children's behaviour and your own feelings about your teaching.

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Notes

About the Author

Margaret began her career as a primary school teacher in Tasmania, Australia and subsequently completed a PhD in Mathematics education at the University of Tasmania. She has spent almost 20 years as a teacher educator and educational researcher in Australia and Hong Kong. She received national awards in Australia for her research on mathematical problem solving and was co-author of *Mathematics for Children*, one of the recent best-selling mathematics education textbooks in Australia. In the past decade, as she has become increasingly aware of the need to develop inner peace and love in the individual, she has developed a growing interest in education in human values. She has authored numerous publications about integrating values education into mainstream teaching, including a book titled *To Teach Not To Punish* (with Anita Devi). In addition, she has authored books and publications addressing the human need for peace, including *Freedom from Loneliness* (with Monika Zechetmayr) and *Place of Tides: Nature's Visualisations for Healing* (with Vics Magsasay). She has travelled widely to study education in human values programmes in Thailand, India, Zambia and Australia. Her particular interest is in nurturing teachers to reflect on their own personal values so that they can become more empowered and empowering in their professional and broader lives. Currently she divides her time between Australia, Hong Kong, Mainland China and India, where she is engaged in voluntary teacher-development projects relating to various aspects of education in human values.

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